Non-Linear Programming for Induced Drafting Practices in Industries

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***Abstract* -The selection of Induced Draft Fan (IDF) characteristics for percolation of gases in various industrial application areas is emphatically contingent upon the filtration rate, particle bed height and the granulatory characteristics of the particles in the system. The permeability of the medium is deterministic factor for selection of IDF capacity and attaining requisite mechanical and chemical properties during process. The present work circumscribes the study of various parameters along with their inter-dependency, using constraint multivariable non-linear optimization technique for the IDF with a typical example from steel industry. The study is first carried out for particle size upto 5 mm, to minimize the pressure drop across the fluidized bed by its optimization against the dependent variables. The investigation is further extended for the particle size upto 10 mm by analyzing the comparative information. The trend thus established, is also validated with the satisfactory fitness for all the considered operational parameters. A normalized dependency for the pressure drop and the shaft power requirement is also presented for the ease of IDF selection for multivariate operational requirements.**

***Keywords* - Induced Draft Fan Permeability, multivariable Optimization, , Fluidized Bed , Under Grate Suction.**

I. INTRODUCTION

Worldwide, induced drafting is extensively exercised for excavation of the exhaust/ waste gases from the system, heat exchange and temperature regulation and to help regulate the rate of process among various applications like steel, power, cement, mining industries. The mechanical drafting machines are responsible for approximately 10% of world's industrial energy consumption. Contrasted to the natural drafts, mechanical induced drafting is established by means of Induced Draft Fans (IDF) and thus my means of a power-drive. Since, induced drafting is implemented in various critical industrial applications posing considerable power requirements, its proper and optimized designing is of prime most concern for designers. In addition to this, entire equipment is tested and designed on various parameters before the final selection various features including a soundproof design, light and simple structure.

In many industrial processes the productivity depends upon the quality of raw material and operational characteristic of a particular plant. The IDF implies not only a substantial cost in a typical process plant but also plays a cardinal role in controlling the process, it becomes more assertive for the designers for proper selection of the IDF. The current investigation is exemplified by a typical process from steel industry for optimization. The dependency of process's mechanical as well as chemical attributes on the input parameters (raw material) poses a unique opportunity to investigate and analyze the optimization practice on a wider range of possibilities. The results obtained are conclusive on optimizing the capacity and obtaining cost effective operational characteristics for IDF, which can further be implemented in various industrial domains also.

II. MATHEMATICAL MODELLING

The agglomerates used for steel production are formed by the heating the raw material and creating suction using ID Fan for the process. The resistance offered by the raw material bed due non-uniform distribution of mixture particle or moisture content, etc., largely decides IDF capacity generating the under-grate suction for the process. The drafting is such adjusted so as to maintain complementary propagation of the combustion and the wave front inside the material bed. The resistance offered by the material depends upon the bed height, rate of filtration through the bed and shape -size of particles. The condition for layer-wise burning and heat generation are distributed as depicted by [1]

$$v = 1.682\ \frac{\in^{1.58}}{(1-\in)^{0.053}}\frac{d^{0.58}\rho_s^{0.53}}{\rho_g^{0.47}\eta_t^{0.053}} \qquad (1)$$

where, v is the minimum fluidizing velocity required through the layer (ms^{-1}), ε is the layer porosity (0.3)[2], d is the diameter of the particles (m) , ρ_s is the density of the sinter raw mix (1800 kgm^{-3}) , ρ_a is the density of the air (1.17 kgm^{-3}) and η_t denotes the dynamic viscosity of the air (1.877 e-6 $kgm^{-1}s^{-1}$)

The calculations are based on the considering the quality of gas (initially air) at ambient conditions (298 K). The minimum velocity for fluidization of the layer (Eq.1) along with the various considered particle size distribution is shown in Table I.

TABLE I

Particle Size (mm)	Fluidizing Velocity (m/s)
0.1 - 0.3	0.160
0.3 - 0.5	0.239
0.5 - 1.0	0.345
1.0 - 3.0	0.616
3.0 - 5.0	0.912

Further, the loss of gas pressure ∇P (mmHg) across a layer of loose packed material can be found out using

$$\nabla P = A\ H\ W^n \qquad (2)$$

Where, W is rate of filtration of gas through particle layer (ms^{-1}), H is the height of the layer (m). The values of the coefficient (A) and the exponent (n) are dependent upon the shape and size of the mixture particles in the sinter bed and are shown below in Table II.

TABLE II

Particle Size (mm)	Coefficient Value (A)	Exponent Value (n)
0.1 - 0.3	6.50	1.16
0.3 - 0.5	3.40	1.30
0.5 - 1.0	1.43	1.39
1.0 - 3.0	0.66	1.51
3.0 - 5.0	0.30	1.77

The analysis is further preceded by considering the following major assumptions –

i. The medium is equivalent to porous media with isotropic distribution of the properties.
ii. Air distribution inside the medium is uniform and voids are connected.
iii. Air behaves as an incompressible fluid with constant thermal properties for entire analysis.
iv. Air is chemically inert for the entire process.
v. No thermal interactions are considered.
vi. No thermo-mechanical losses are accounted for performance mapping of ID Fans

III. OPTIMIZATION APPROACH

The agglomerate raw mix bed composes of fine sized ore particles, which are fluidized by gas-filtration generated by means of under-grate suction. For further analysis the mathematical formulations are presented here in-order to study the minimization of the pressure losses incurred by the gas across its filtration through the mixture-bed.

Mathematically, the objective can be approached as [4]

$$\min \ \nabla P(H,W)$$
$$\forall \ (H,W) \in R^2$$

$$D = \left\{ \begin{matrix} a = (H,W) \parallel \ \mathrm{H}min < H\mathrm{max}, \\ \mathrm{Wmin} < W < W\mathrm{max} \end{matrix} \right\} \qquad (3)$$

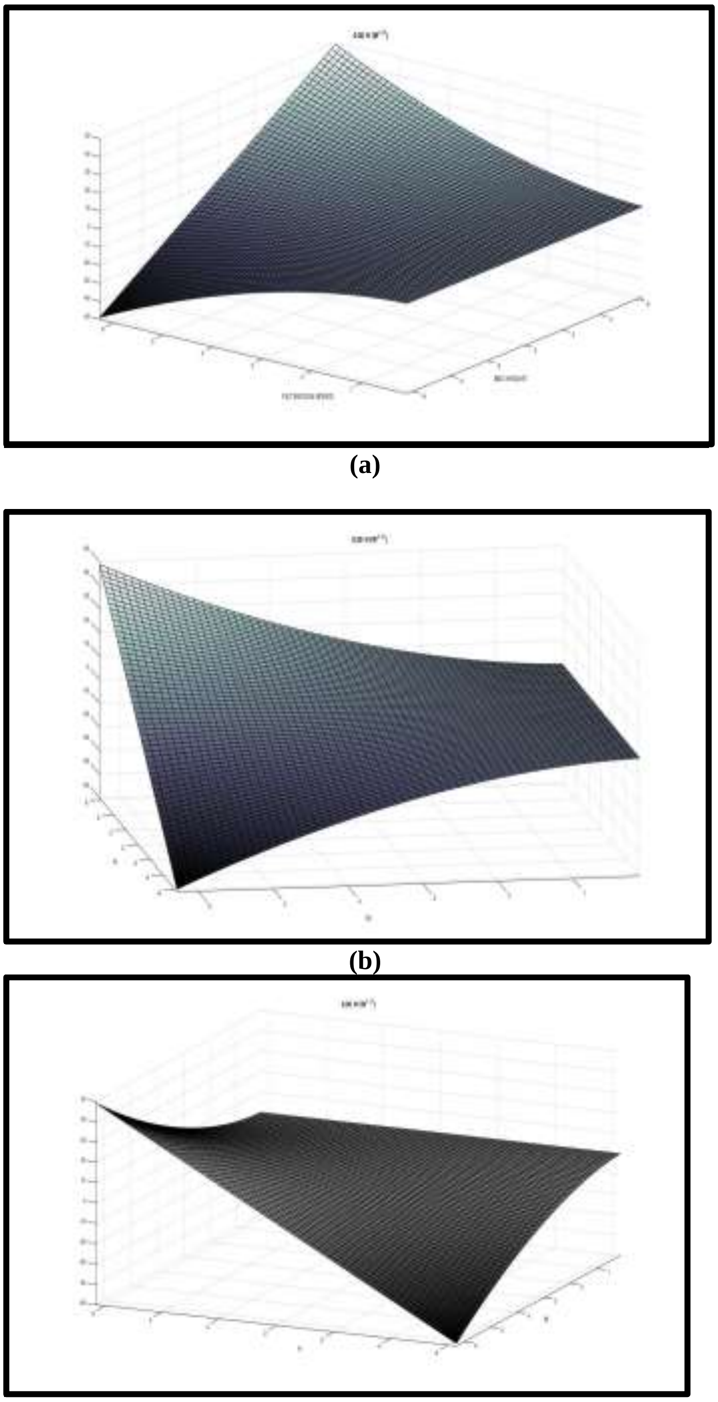
Where "R" denotes the Euclidian Space, "D" is the interest domain and "a" denotes the admissible set of values for Height of the bed (H) and Filtration Rate (W).

Since the pressure drop is also dependent upon the height of the medium, in this analysis it is assumed to be restricted within the range of 50 mm to 2000 mm, for all practical purposes. The height of the particle bed is restricted to lowest value of 50 mm considering the minimum requirement of the hearth layer for the proper operation of the sinter machine. The lower bound of the filtration rate (W) is set by the minimum gas-flow rate required to fluidize the mixture layer (v), as represented in Table1. Similarly the upper limit of the filtration rate is set to be 5 ms^{-1}, where the gas in its movement through sinter bed incurs a substantial static pressure loss.

It can be easily observed that the considered objective function bears a direct dependency on its variables. The variables considered for mathematical treatment are considered within a range and are selected as bound variables. Hence, the non-linear function is said to be dependent upon two bound variables such that

$$50 < H < 2000$$
$$v < W < 5 \qquad (4)$$

A three dimensional physical interpretational representation of the pressure drop function (particle size 0.3-0.5 mm) is shown in Fig 1. Also, it can be established that the local minima of the function is same as the global minima in the analytical domain. This unique property of the objective function helps in reducing the computational cost to obtain a smooth convergence. Also the congruency in equivalence of the local and global minima helps in employing the gradient-based methods for the optimization, which would otherwise be not applicable.



(a)

(b)

(c)

Fig.1: Three Dimensional physical representation of pressure drop of air against the height (H) and filtration rate (W) of air across sinter bed. (Particle size 0.3-0.5 mm).

IV. RESULTS AND DISCUSSIONS

The pressure drop across various layers of mixture (raw-material) defines the requisites for the selection of operational and performance characteristics of induced draft fan. As the agglomeration is attained along the length of the bed, the effective particle size also varies due to the metallurgical changes inside the particle raw material layer. For analysis the agglomerate mixture can be considered analogous to a porous media. The pressure drop across a porous medium depends upon the inertial and the viscous resistance offered by the medium to the flow. The inertial forces are related to the rate of filtration of the liquid phase (gas) and the viscous resistance is affected by the size of the particles comprising the medium.

In various industrial processed the porosity is an important parameter that significantly affects the properties of the products. For iron ore agglomeration it is observed that the pore diameter needs to be larger than 0.01 micrometer for the gas to have sufficient access to the pores. When the micro pores are coalesced to pores of a size of more than 1 to 5 micrometer, the specific surface area of the sinter is decreased and so does its reduction. Elimination of the coalescence of micro pores and increase of the number of small pores make it possible to increase the surface area of the sinter and obtain a substantial improvement in its reducibility [5]. The Pressure loss function as depicted in Eqn.2 can be shown with optimized relation to various particle sizes in Table III.

TABLE III

Particle Size (mm)	Optimized Pressure drop (Pa)
0.1 - 0.3	5171.190
0.3 - 0.5	3526.098
0.5 - 1.0	2171.719
1.0 - 3.0	2116.947
3.0 - 5.0	1699.064

The method adopted here is based on Non-Linear multivariable constraint numerical optimization technique. The codes are implemented in the MATLAB numerical environment. The Active Set algorithm searches for the minimum of scalar function (Pressure Drop) based on initial guesses [6]. These constraints on the function are passed as the arguments subjected to the equalities and inequalities of the bound variable. The code explicitly uses the finite difference approximation instead of calculating Hessian matrix using quasi –Newton approximation.

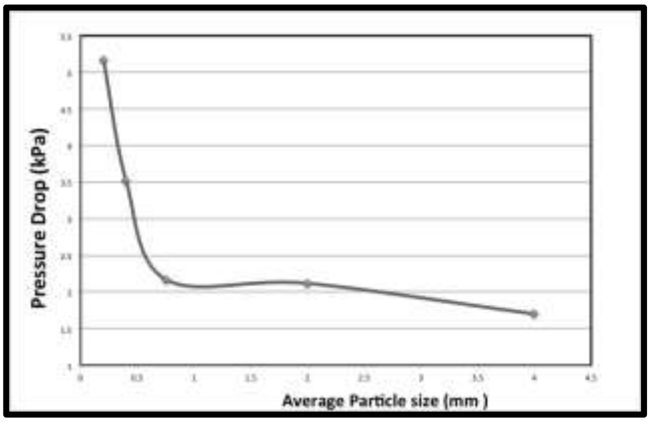


Fig 2. Pressure drops of waste gas against various particle sizes

The finite central difference scheme is used for estimation of the gradients to enhance the accuracy of the analysis [7][8][9]. The tolerance for the constraint violation is set as 1e-6. Termination tolerance on inner iteration Sequential Quadratic Programming constraint violation is 1e-6.

As discussed previously also, the range of the bound variables are based on their practical utility in their respective applications. search for the local minima is based on the Line search methodology with the directional-derivative calculation performed for each step of the iterations. A forward difference finite derivative is calculated for each step of pressure drop optimization calculations. Since this algorithm is gradient-based method, the first order derivative continuity of the objective function as well as the constraint function is mandatory pre-requisite of the calculations.

As for the pressure drop is in direct proportionality to its dependent variables, and the local minimum achieved is also the global minima (Fig 1). A detailed iterative calculation for specific cases is also shown in Appendix1.

Line search method is an iterative search method that utilizes a direction of search d_k , a scalar step length α^* , and the value of the current iterate x_k , in order to find the next iterate x_{k+1} , such that[6]

$$. x_{k+1} = x_k + \alpha^* d_k$$

At each step of the algorithm, the line search method searches along the line containing the current point x_k , parallel to the search direction , which is the vector determined by the algorithm. The Line search method attempts to decrease the value of the objective function along the direction by repeatedly minimizing the polynomial interpolation models of the objective function. The line search methodology is based on two major steps

i. The bracketing phase of the method determines the range of the points on the line to be searched.
ii. The sectioning phase divides the brackets into sub-intervals, on which the minimum of the objective function is approximated by polynomial interpolation.

The acceptable points for the method require that the step length should sufficiently decrease the objective function and should be of considerable finite size.

It is evident from Fig. 2, that the pressure losses are substantially high for the particle size less that 0.01-micrometer. These losses increase in an approximate exponential manner for even minute reduction in the agglomerate particle size. However, the losses tend to decrease with increase in the particle size. But, such conditions are not favorable for better reduction of the mix by the gas.

EXTENDED OPTIMIZATION

The previous study has established the importance of granulatory characteristics of the particle distribution to attain the desired outcomes. Hence, the approach is further extended to estimate the optimum pressure losses to be incurred by the gas for various particle size given similar operational conditions as previously discussed by maintaining similar productivity aspects.

Fig. 3 is obtained by extrapolating the trend observed in Fig. 2 for the variation of optimized (minimized) pressure loss against various sized particles. Observing the best-fit function for the calculated values of pressure loss sets up the trend, it can also be extended to any size of particles.

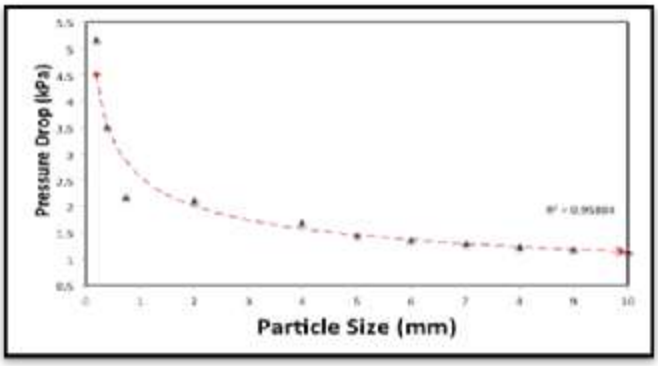


Fig. 3. Optimized pressure drop for the gas against various particle sizes

Further, Fig.4 inherits the relationship among one of the most important aspect of the sintering process. Permeability of the medium offers the viscous resistances for the filtration of the gas. The permeability of the sinter raw mix bed can be calculated as

$$K = \frac{d^2 \in^3}{150\,(1-\in)^2} \qquad (5)$$

Permeability (m^2) can be observed to have logarithmic (base 10) proportional dependency on the media porosity and average size of the particle (m) in Fig 4. As expected, it increases with increase in the particle size of the medium, due to availability of higher void fractions

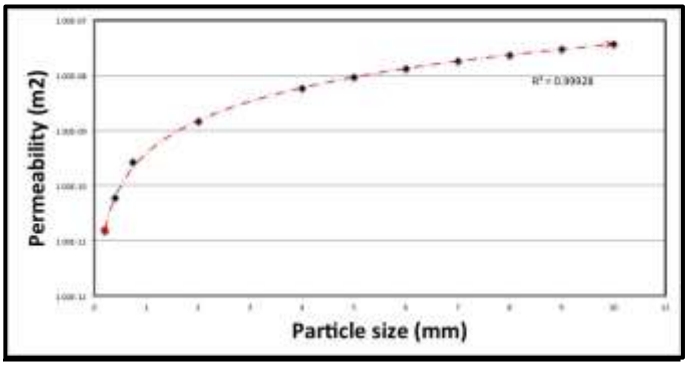


Fig. 4 : Variation of permeability of media with the particle size.

Fig. 5 is obtained by considering the minimum fluidizing filtration rate of the gas through the various layers of the particle mixture bed. It depicts the logarithmic (base 10) relationship of viscous resistance parameter (Permeability) with the optimum pressure drop. It can be seen that the pressure drop increases with the decrease in the permeability of the medium. This decrease in permeability is caused by decrease in size of sinter mix particles, thus increasing demand of the under-grate draft suction for process.

Table IV shows one of the most important outcomes of this study. Since the pressure drop is optimum for the global minimum of the bed height, i.e. 50 mm we can extend the same concept for the selection of the ID fan capacity for various bed heights as per the sinter machine operational requirements.

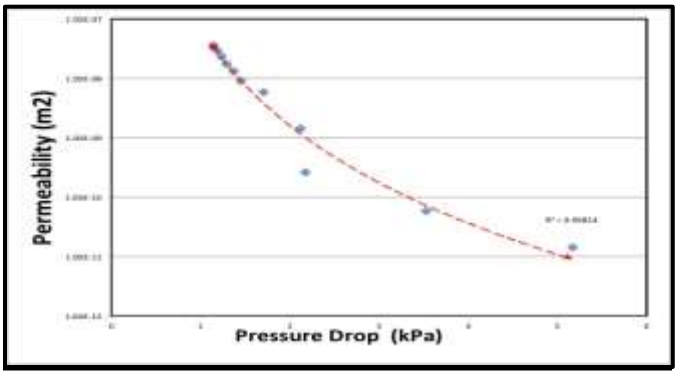


Fig. 5 variation of the Permeability with optimized pressure loss

The results are summarized in Table V also. The indicated results can be extrapolated or interpolated for various filtration rates and granulatory features of the mixture bed.

Further, comparative information is depicted in Figure 6, with the optimum pressure drop for a bed height of 1000 mm and the shaft power requirement of the induced draft fan drive for 100 m^3s^{-1} filtration rate of the waste gas through bed. The information is presented in normalized form to enhance the applicability of the observations for multi-varied operational conditions. It should also be noticed that the shaft power requirement of the drive is not considered with any other kind of losses that are electro-mechanical in nature.

TABLE IV

Average Particle Size (mm)	Optimum Pressure-drop (Pa)	Permeability (m^2)
0.2	5171.19	1.47E-11
0.4	3526.098	5.87E-11
0.75	2171.719	2.66E-10
2	2116.947	1.47E-09
4	1699.064	5.88E-09
5	1455.129	9.18E-09
6	1364.925	1.32E-08
7	1293.035	1.80E-08
8	1233.83	2.35E-08
9	1183.86	2.98E-08
10	1140.88	3.67E-08

The utility of formulation is also remarkable to understand divergence in chemical and mechanical behavior of domain in terms of mapping the average size of the particles at any given instant of time during process.

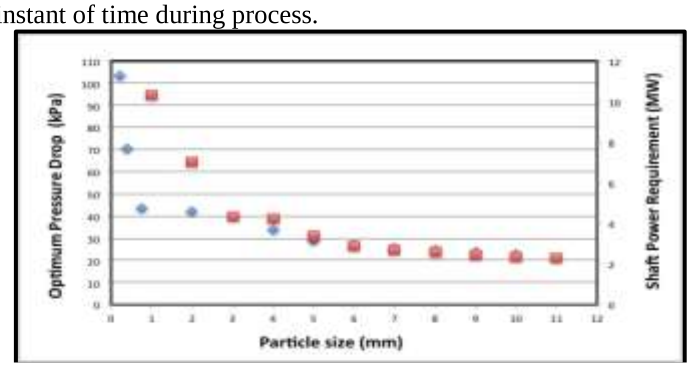


Fig 6: Normalized relationship of optimum pressure drop and motor shaft power requirement

TABLE V

Average Particle Size (mm)	Optimum Pressure Drop (kPa)	Shaft Power Requirement (MW)
0.2	103.4238	10.342380
0.4	70.52196	7.052196
0.75	43.43438	4.343438
2	42.33894	4.233894
4	33.98128	3.398128
5	29.10258	2.910258
6	27.2985	2.729850
7	25.8607	2.586070
8	24.6766	2.467660
9	23.6772	2.367720
10	22.81758	2.281758

It is also worthwhile to note that this study has been carried out without considering the thermo-mechanical losses i.e., like the duct losses for the waste gas flow at high temperature, windage losses for the centrifugal fan, motor and shaft bearing inertial losses etc.

V. CONCLUSIONS

The present investigation provides an insight for identifying the most prominent factors affecting the selection of the ID Fan for multivariate industrial applications. The optimization carried out in the study emphasizes and establishes its needs in achieving a breakeven for high efficiency and low drive requirement during operation. The results are also successful in explaining the employability of the constraint non-linear optimization techniques in along with their limitations in analyzing the importance of granulatory characteristics of raw mix along with the bed height and filtration rate for choice of optimum pressure drop and drive shaft requirement respectively. Also, the reported normalized results can easily be extrapolated for any desirable process requirements, except for the to achieve a high productivity and yield.

REFERENCES

1. Schenk, H., W. Wenzel and H. D. Butzmann, *Verhindern des Zusammehsinterns von Wirbelbettenbei der Reduktion von Eisenerzen*, Archly fur das Eiseenfuttenwesen, 33,211-216 (1962)
2. Komarov S. V. and Kasai E., *Simulation of Sintering of iron ore bed with variable porosity*, Phoenix User Conference, Melbourne, 2004
3. Zinov'ev Yu. I., *Drag of a blocked fluidized bed*, Fluid Dynamics, Sept 1977, vol. 12, issue 5 ,pp. 790-794
4. Rao, S. S., *Engineering optimization: Theory and practice*, 3rd edn., New Age New International, Delhi,2013.
5. Langston, B. G. and F. M. Stephens, Jr. ,*Self-Agglomerating Fluidized-Bed Reduction*, J. Metals, 4,312-316 (196).
6. MATLAB Documentation, Release v2014b, The MathWorks Inc., USA
7. Gill, P.E., W. Murray, and M.H. Wright, *Practical Optimization*, London, Academic Press, 1981.
8. Han, S.P., *A Globally Convergent Method for Nonlinear Programming*, Vol. 22, Journal of Optimization Theory and Applications, p. 297, 1977.

APPENDIX 1

Average Particle size 4 mm

Iteration	F-count[i]	f(x)[ii]	Max constraint	Line-search step-length	Directional Derivative
0	3	30	-0.088		
1	6	25.4102	0	1	-15.2
2	9	24.6469	0	1	-2.55
3	12	21.3991	0	1	-2.55
4	15	12.7433	0	1	-2.55

Average Particle size 2 mm

Iteration	F-count	f(x)	Max constraint	Line search step-length	Directional Derivative
0	3	66	-0.384		
1	6	31.5453	0	1	-50.7
2	9	31.1666	0	1	-0.318
3	12	26.1247	0	1	-0.318
4	15	21.0828	0	1	-0.318
5	18	15.8775	0	1	-0.318

Average Particle size 0.75 mm

Iteration	F-count	f(x)	Max constraint	Line search step-length	Directional Derivative
0	3	143	-0.655		
1	6	32.1107	0	1	-84.1
2	9	31.5783	0	1	-0.326
3	12	16.2883	7.105e-15	1	-0.326

Average Particle size 0.40 mm

Iteration	F-count	f(x)	Max constraint	Line search step-length	Directional Derivative
0	3	340	-0.761		
1	6	51.0944	0	1	-99.9
2	9	48.5262	0	1	-0.529
3	12	26.4464	0	1	-0.529

Average Particle size 0.20 mm

Iteration	F-count	f(x)	Max constraint	Line search step-length	Directional Derivative
0	3	650	-0.84		
1	6	72.5277	-1.388e-16	1	-103
2	9	68.7575	0	1	-0.776
3	12	38.7849	0	1	-0.776

[i] Count based on step size of 3 units
[ii] f(x) denotes the value of pressure drop function

