Reduction Of Quasi-Periodic Noise From Digital Gray Level Images Using Probabilistic Approach

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Abstract: **This paper manifests an efficient algorithm to minimize the effect of periodic as well quasi-periodic noise from digital images. The algorithm uses a-contrario method for the detection of noise spikes in frequency domain. The performance of this method is tested on the different test images. The results of this method is compared with the algorithm to remove quasi-periodic noise based on mathematical parameter and found that the proposed method gives better results than other existing methods and also can be apply for any type of digital images.**

Keywords: **quasi-periodic noise, a-contrario.**

Introduction

Digital image is basically a two dimensional discrete signal. The smallest element of the image is called pixel. To transform a digital image from spatial domain to frequency domain, discrete Fourier transform is used.

The 2-D discrete Fourier transform of an $M * N$ image $f(x, y)$ is denoted by $F(u, v)$ is given by

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(\frac{ux}{M}+\frac{vx}{N})}$$

For $x = 0,1,2,3, \ldots., M-1$ and $y = 0,1, \ldots, N-1$

Inverse operation is done by following equation:

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(\frac{ux}{M}+\frac{vx}{N})}$$

For $u = 0,1,2,3, \ldots., M-1$ and $v = 0,1, \ldots, N-1$

Figure: 1(a) Original Gray level Lenna Image

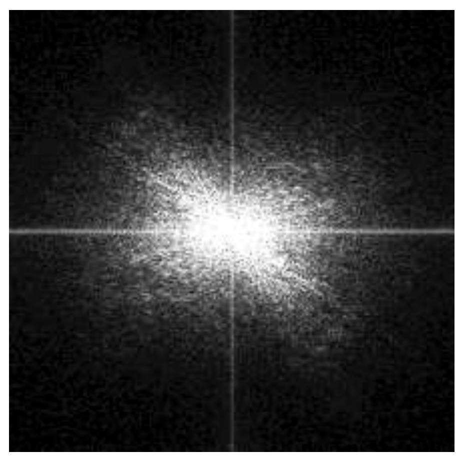


Figure: 1(b) Magnitude Spectrum of Lenna Image

On observing the magnitude spectrum shown in Fig. 1(b) of the Lenna image shown in Fig. 1(a) it concludes that most of the information of a digital image presents in the low frequency region. The horizontal and vertical lines present in the magnitude spectrum belong to the discontinuities present at the left/right and top/bottom borders of the Lenna image respectively. It can be remove from the magnitude spectrum by multiplying a 2 dimensional Hann window with the same width as of the original image.

As there are a variety of noises existing in digital images during image acquisition or transmission which affect the digital image processing differently.

These noises are de-noised either in spatial domain or in frequency domain. Most of the noises have statistical parameters for e.g. PDFs, PSDs etc. Hence to detect them and eliminate their effect is easier than periodic and quasi-periodic noises which affect the digital images in the form of spurious repetitive patterns covering the entire image in addition quasi-periodic noise changes its behavior in spatial domain. Hence to detect and eliminate it is more difficult than others. Detecting them in frequency domain is easier than in spatial domain because they appear in the form of spikes in the frequency domain at certain discrete frequencies that's why mostly researchers manifested their algorithm to reduce them in frequency domain.

As it was observed that most of the image information lies in the low frequency region hence the algorithm should take care of it so that during noise elimination useful component of the image could be removed hence loss of quality.

Some existing methods require expert tuning [1] [2]. Thus the problem is to automate spike detection. The proposed algorithm employed a-contrario method to detect spikes in frequency domain. A-contrario method was given for the detection of alignments in images [3] and proves to be well adapted to many image analysis tasks as Gestalt grouping [4] [5], detection of moving objects in videos [6], line segment [7] or elliptical arc [8] detection. In this detection method, concept of meaningful feature is introduced. A feature is meaningful among all the features if it is not likely to be caused by the background process. To decide whether a feature is meaningful or not is based on Number of False Alarms (NFA) which corresponds to the average number of such a feature expected from the background process since meaningful features are not likely to be caused by background process hence false alarm [9].

Suppose features have real values and meaningful features are not likely to have a high value say x and meaningful features are sought among N features in the background process then the NFA for x is

$$NFA(x) = N \Pr(X \geq x)$$

Where X is a random feature

The value of $\Pr(X \geq x)$ is calculated from statistical laws which are either parametric or theoretically estimated. In most of the papers meaningful features are those for them $NFA \leq 1$ or $\log NFA \leq 0$ i.e. negative. In this sense a-contrario detection is a parameter free method.

2. Literature Review

[10]Automated removal of quasi-periodic noise using frequency domain statistics

Since it is known that the expected power spectrum of natural images can be modeled by a function which decreases with the inverse of some positive power of frequency [11] (i.e. $\widetilde{|\mathrm{P}|}^2 \propto \frac{1}{\mathrm{f}^{\alpha}}$ where $\mathrm{f} = \sqrt{\mathrm{u}^2 + \mathrm{v}^2}$) and in every small patches extracted from the image (corrupted with periodic noise) has only periodic pattern caused by periodic noise itself. Using these results Frederic SUR gave an algorithm to automatically detect the position of noisy component in frequency domain only in the natural images and after detecting the position of noise component eliminated them.

Algorithm: 1

Given an image I of size $\mathrm{M} * \mathrm{N}$ corrupted by quasi-periodic noise, the algorithm has following steps:

1-Extract patches of size $\mathrm{L} * \mathrm{L}$ from the original image keeping the size of patches enough large to ensure both a good accuracy in the periodic noise detection and detect-ability of low frequency noise but not too large, so as to make it possible to build enough independent patches from the noisy image of interest.

2- Now find the power spectrum of each of the patches and finally average them to get average power spectrum of all of the patches denoted as $\widetilde{|\mathrm{P}|}^2$.

3- Now fit the power law distribution to the average power spectrum as calculated above. Fitting is done by robust linear regression between the frequencies f_0 and f_1 cycles per pixels where $(\mathrm{f}_1 > \mathrm{f}_0)$. Fitting results gave values of unknown parameters A and α and also value of σ (standard deviation).

$$\log[\widetilde{|\mathrm{P}|}^2(u,v)] = A - \alpha \log(u,v).$$

4- Now using 3σ rule (where it represents upper limit in the average power spectrum from the Red line as shown in the figure: which was obtained by fitting the power law on to the average power spectrum in above step) noise spikes are differentiated to the image components as follows:

If

$$\log[\widetilde{|\mathrm{P}|}^2(u,v)] > [A - \alpha \log(u,v)] + 3\sigma$$

Then at that discrete frequency which satisfies above condition will be a noise spike. Hence it gives a matrix of only 1 and zeros say $M_0$where'1'represent presence of noise at that frequency.

We restrict to apply this condition for frequencies $f < f_2$ because low-frequencies don't correspond to repetitive pattern.

5- Now interpolate M_0 of size L*L to size M*N and by multiplying spectrum of original image with 1- M_0eliminate the effect of quasi-periodic noise hence (1- M_0) acts as notch filter.

an a-contrario approach to quasi-periodic noise removal

This approach is totally probabilistic since noise is random in nature and its behavior could not be predicted. The main concept of this is explaining with the help of figure 2 as shown below.

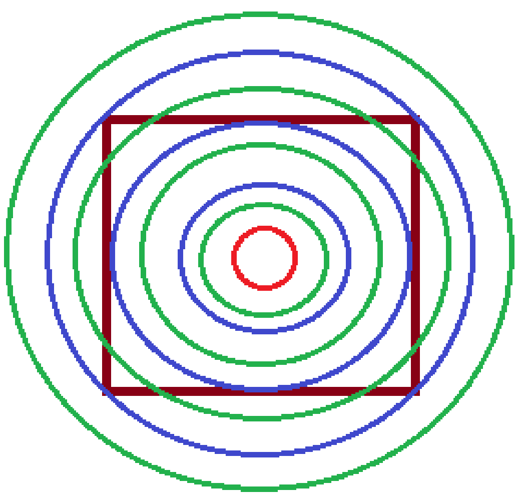


Figure: 2 Circles representing number of regions in the image shown as square shaped box

The brown colored square box is representing the size of Fourier transformed digital image. The smallest red colored circle is representing the low frequency region inside it hence algorithm to find noisy component should not apply inside this region whether it should start from the boundary of the circle. Each corner of the square contain maximum frequency component of this image. Hence divide the region between maximum frequency component and low-frequency component represented as red circle into the number of regions as shown in figure 3 in the form of blue rings. The first region start from red circle and ended on the first blue circle from where next region is starting. Similarly some rings of green color are also viewing in the figure 3. The significance of them is to accumulate neighboring power spectral density components for comparison to the minimum power spectral density components belonging to the blue colored rings. The first ring belonging to neighboring components starts also from the red circle and on the 2nd green circle. The next ring starts from the first green circle and end on the 3rd green circle and so on.

Algorithm: 2

Given an image I of size X $*$ Y corrupted with quasi-periodic noise, steps to reduce it are as follows:

1- Extract patches each of size L $*$ L from the original image I. Patch size should be large enough but not so large that patches would be correlated hence independency of the patches should be maintain. As it was found experimentally that a good compromise is to take a sampling step of L/8 in both horizontal and vertical directions, which gives a total number of patches equal to

$$\left\lfloor\frac{8(X-L)}{L}\right\rfloor * \left\lfloor\frac{8(Y-L)}{L}\right\rfloor$$

Where $\lfloor . \rfloor$ denotes floor operation.

2- Calculate the power spectral density of each of the patches individually and find out element-wise minimum power spectral density from all patches

3- Now the problem is to find out noisy components from non-noisy one in the minimum power spectral density. Compare each minimum power spectral density component one by one to their neighboring frequency components cumulated from the power spectral density components from each patches represented as green rings in figure 3. As it is known that most of the information lies in the low frequency region hence this process will start from frequency$(f_2 = \frac{8}{L})$.Hence evaluate for each patch:

$F_R = Probability$ (PSD of all neighboring components including the components present at the same frequency in PSD spectrum of the patch at which minimum PSD component present ≥ Minimum PSD of a component).

As to be noisy component of that minimum PSD component the above condition must be failed most of the time. Hence $F_R$should be less but how much to solve this problem a threshold limit is decided and a new parameter is calculated as:

NFA (number of false alarms of each minimum PSD component) = (Total number of Blue colored Rings)*(Number of component in each ring)*(total number of patches)*F_R

Now if NFA $\leq$ 1then minimum PSD component belongs to that particular frequency would be noisy.

4- Hence detection of frequency corresponding to noisy component has done. Now eliminate that particular frequency component using notch filter.

3. Results and Analysis

The two dimensional periodic noise, $\eta(x, y)$ which is given by the below equation, is added to a sample of $512 * 512$ Grey level Lenna test image and the algorithm 1 is applied.

$$\eta(x,y) = A * \sin\left(2 * pi * \frac{f1}{M} * x\right) .* \sin(2 * pi * \frac{f2}{N} * y)$$

Where M and N are the size of the image, A is noise amplitude and $f1$, $f2$ are the frequencies in x and y axis respectively.

Performance of both of the above mentioned approach is tested on same standard test Lenna image of size $512 * 512$ and results are shown below.

Results of algorithm 1:

Synthetic periodic noise is added on original Lenna image of size 512*512 as shown in figure 3a).

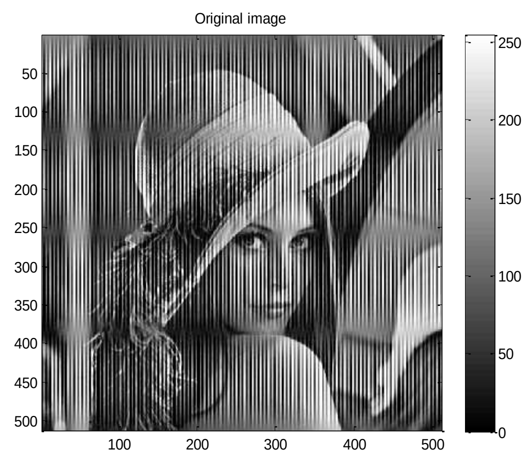


Figure: 3a) Gray level Lenna image corrupted with quasi-periodic noise

This synthetic periodic noise has amplitude $A=150$ and $f1$ 100 and $f2$ =2.

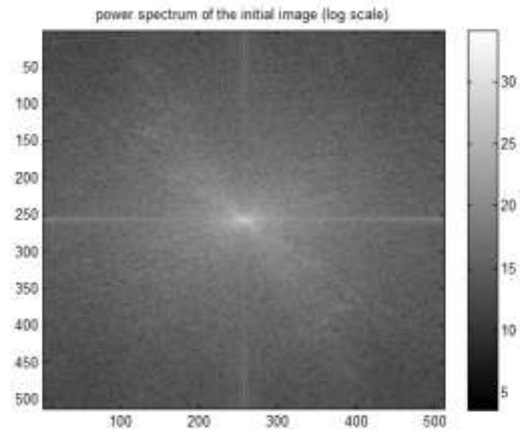


Figure: 3b) Power spectrum of the Lenna image

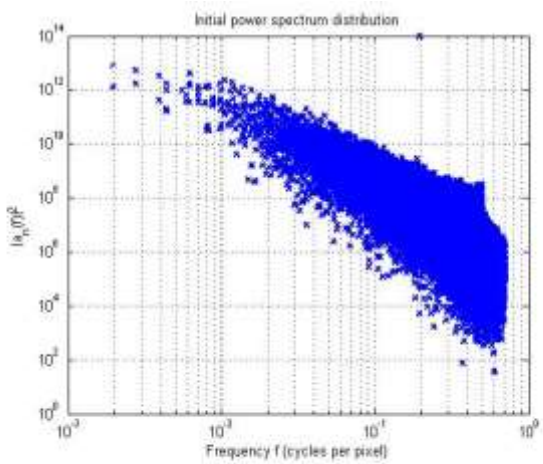


Figure: 3c) logarithmic plot of PSD Vs frequency

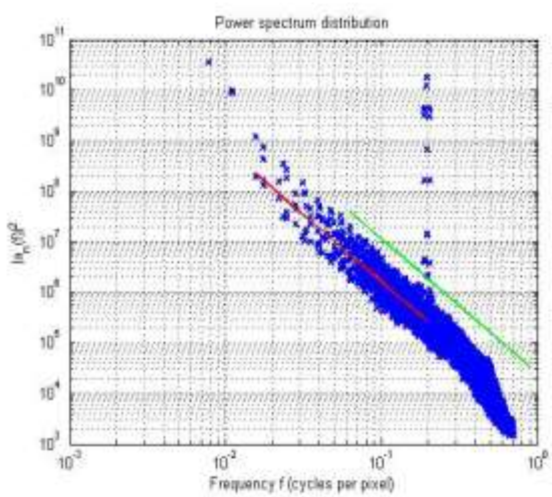


Figure: 3d) logarithmic plot of Average PSD Vs Frequency

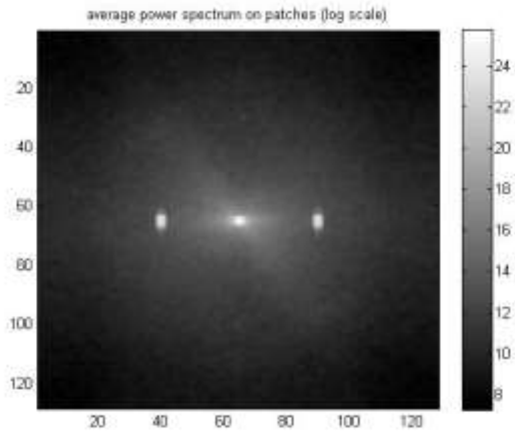


Figure: 3e) Average Power spectrum

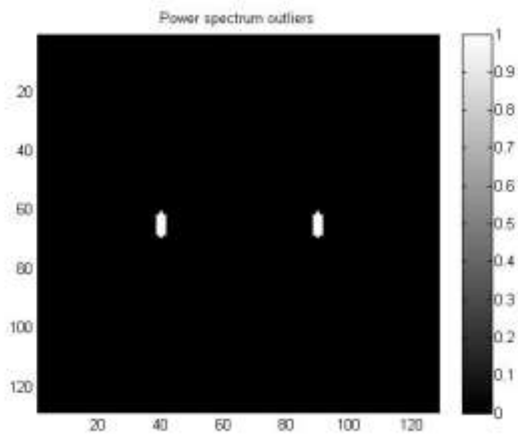


Figure: 3f) Outliers Map of Power spectral density

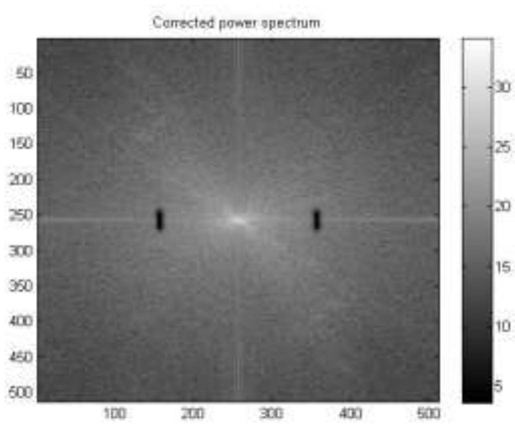


Figure: 3g) Corrected power spectrum

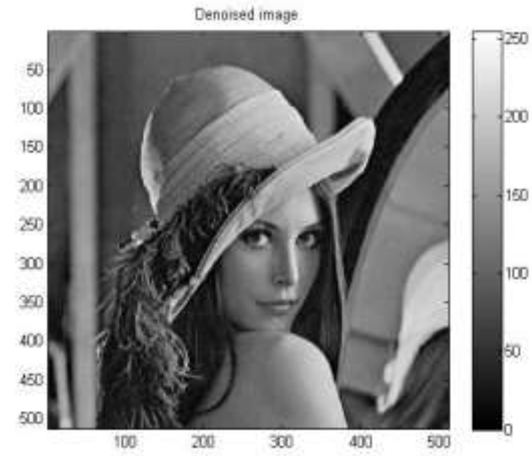


Figure: 3h) Denoised original image

Hence above mentioned approach corresponds to algorithm 1 is removing quasi-noise well qualitatively as seen both corrupted and denoised Lenna image.

Results of algorithm 2:

Same synthetic periodic noise is added into the same Lenna image and algorithm 2 is applied to de-noise it and as shown in the results qualitatively it is also giving good results than other existing older methods [12] [13][14]. Noise components in spatial domain are also clearly extracted. All the results are shown in figure 4.

PSNR can be improved if the size of region corresponding to the neighboring components is increased that means finding the NFA of any particular component by comparing more number of neighboring components than previously because in probability it is better to perform any event a large number of times so that probability of happening that event at once can be commented more accurately.

This algorithm is applied also on the Barbara test image as shown in figure 6 a) (without adding synthetic noise) as Barbara contains high intensity textures found in the cloth used in pant, tie, scarf which is the part of image and it is found that for patch size L= 256 it gives MSE(mean squared value) zero and hence PSNR infinity means it is preserving this information as it is.

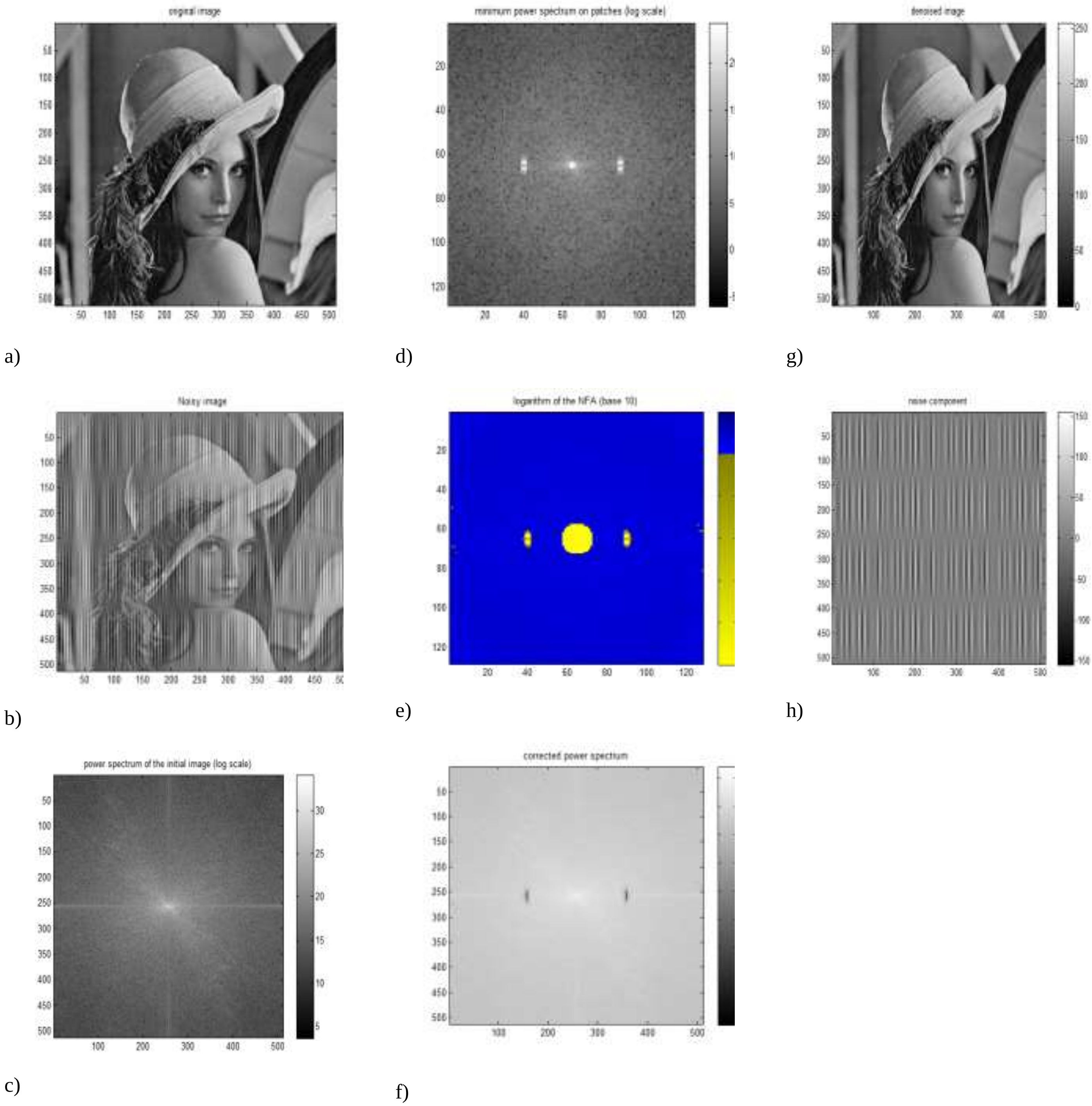
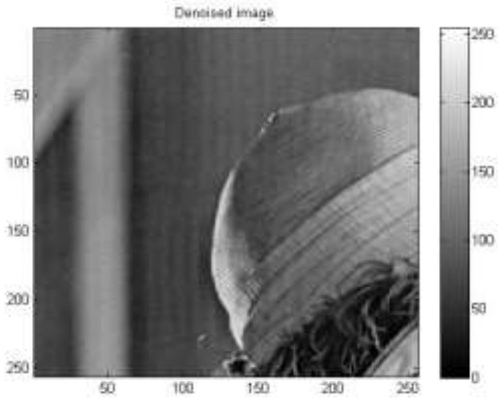


Figure: 4 a) Original image of size 512*512; b) Corrupted image; c) Power spectrum; d) element-wise minimum power spectrum; e) Map of NFA f) Corrected power spectrum; g) De-noised image; h) Retrieved noise components in spatial domain.

Comparison of algorithm 1 and algorithm 2



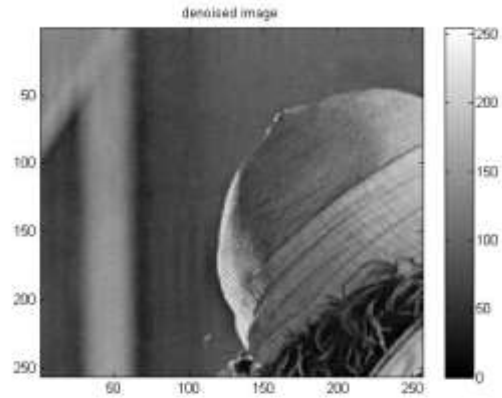
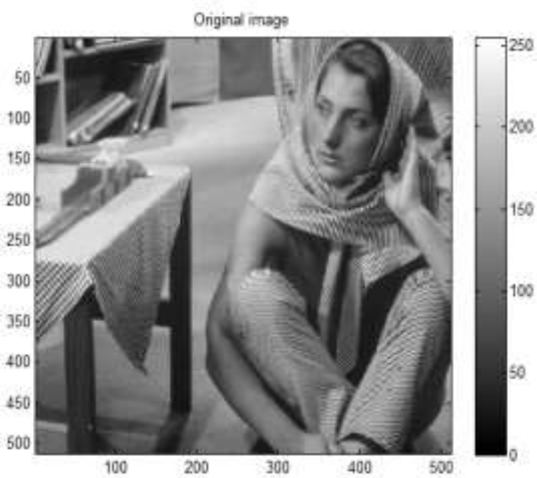


Figure: 5a) Zoomed portion of denoised lenna image using algorithm 1

Figure: 5b) Zoomed portion of denoised lenna image using algorithm 2

These results are taken by adding low frequency (*f1*=50, *f2*=2) synthetic periodic noise with A=150 and as shown in the figure 5a) and b) it is clearly visible that denoised image using algorithm 1 has more vertical stripes than using algorithm 2 and PSNR (peak signal to noise ratio) of image shown in figure 5a) is 40.0390db whether that shown in 5b) has psnr 42.7923db. Hence qualitatively and quantitatively both algorithm 2 gives better results particularly at low frequency than algorithm 1.



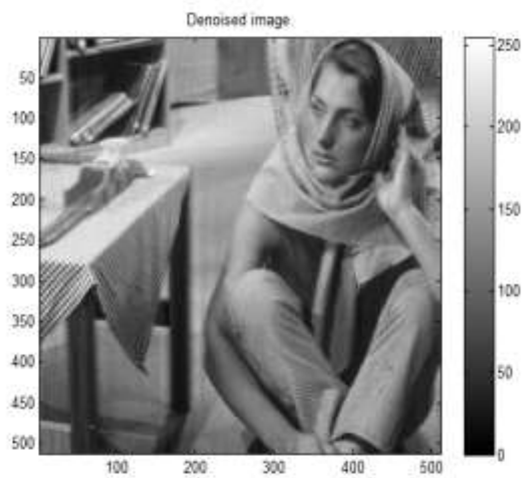


Figure: 6a) original Barbara test image

Figure: 6b) Processed image through algorithm 1

Here as it is clear from the Barbara test image it has high intensity high frequency components hence to check whether which one will give best result in such case original image was processed through algorithm 1 and 2 without using synthetic periodic noise and it was found that algorithm 2 doesn't affect it anyhow but algorithm 1 removed its high intensity high frequency components hence algorithm 2 is accurately performing for all cases whether algorithm 1 is only for natural images and also for low frequency it doesn't perform well.

Conclusions

Since method 2 is parameter free method hence it gives better result always than other existing methods [12] [13] [14] also it provides highest PSNR than PSNRs of other existing methods. Also PSNR should be increase by increasing the region corresponding to neighboring components since as in probability theory it gives better result when there is more number of elements. Also by increasing the size of the patch (L) till the certain value PSNR can be increased.

Future Work

1. In the future work de-noising can also be done with de-blurring of images.
2. Quasi-periodic noise with mixed noise leads to the severe degradation in images, this can be further investigated and validity of the algorithm can be re-evaluated.
3. Denoised image test can be done under various metrics like SSIM (structural similarity index) etc.
4. Experiment can be performed on satellite images which suffer from quasi-periodic noises.

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