Different Methods of Construction of the Optical Orthogonal Codes: Overview

Amita Asthana[1] and **Prof. P. K. Chopra**[2]

Department of Electronics Communication Engineering, Ajay Kumar Garg Engineering College, 27Km stone, NH-24, Ghaziabad. 201009 UP

[1]amitasrivastava15@rediffmail.com, [2]pradeepkchopra@gmail.com

Abstract- **An optical orthogonal code is a family of (0, 1) sequences and possesses good auto and cross correlation properties. It finds application in a code division multiple access fiber optic channel. The applications are also in the areas of mobile radio, spread spectrum communications, and radar and sonar signal design. This paper describes all the schemes to design the unipolar codes characterised by a general parameter (n, w, λ).**

Keywords: Orthogonal codes, Auto-correlation properties, Cross-correlation properties, ON-OFF keying, Spread spectrum communications.

I. INTRODUCTION

Orthogonal codes find extensive applications for efficient Optical communications. There are various methods to construct optical orthogonal codes (OOC).

If (n, w, 1) is the optical orthogonal code C with M codewords set, then system can work with M transmitters simultaneously. Each transmitter is assigned a codeword. At the transmitter, every information bit is encoded into a frame of n optical chips. If the transmitter bit is 1 then total frame will be passed by the transmitter using ON OFF Keying according to the codeword but if the information bit is zero then no light pulses are sent in the corresponding codeword.

The (0, 1) sequences of an OOC are called its codewords. The size of an OOC denoted as $|c|$, is the number of codewords in it. Cyclic shifts of codewords do not affect its correlation properties. If c' is derived from c by shifting an arbitrary subset of codewords, then c' is still an (n, w, λ_a, λ_c) code.

For a given set of values of n, w, λ_a, λ_c, the largest possible size of an OOC is denoted by φ (n, w, λ_a, λ_c).

We may also view optical orthogonal codes from a set theoretical perspective. All M users transmit at any time. There is no network synchronization required. At the receive end, correlation–type decoders are used to separate the transmitted signals. The decoder consists of a bank of M tapped delay lines, one for each codeword. The delay taps on a particular line exactly match the signature sequence. These tapped delay lines can be easily implemented, which actually calculate the correlation of the received waveform with its signature sequence. Since the correlation between different signature sequences is low, so delay line output is only high when the transmitter information bit is 1, otherwise the whole frame duration will be a complete off.

II. PERIODIC CORRELATION AND APERIODIC CORRELATION PROPERTIES

Here we consider the concepts of correlation properties. Larger codes can be designed if only aperiodic correlation properties are required. If an ideally orthogonal set has perfect aperiodic correlation properties, the code set will also have a perfect periodic correlation properties. If we can use aperiodic correlation functions to study the spreading code set then we can greatly reduce the overall complexity of the analysis.

III. ONE DIMENSIONAL UNIPOLAR ORTHOGONAL CODES

The binary sequence of length n bits and weight w along with all (n-1) circular shifted sequence represent the same code. For length n and weight w one or more than one unipolar orthogonal codewords can be generated in the same orthogonal set. Suppose two codewords X and Y are selected from the same orthogonal set;

$$X = (x_0, x_1, x_2, \ldots\ldots, x_{n-1}) \tag{1}$$

$$Y = (y_0, y_1, y_2, \ldots, y_{n-1}) \; ; \quad x_t, y_t \in (0,1) \tag{2}$$

$$\lambda_a \geq \sum_{t=0}^{n-1} x_t x_{t \oplus \tau} \tag{3}$$

$$\lambda_b \geq \sum_{t=0}^{n-1} y_t y_{t \oplus \tau} \tag{4}$$

For $\tau = 0$, λ_a is equal to weight w, which is auto-correlation peak .

Further,

$$\lambda_c \geq \sum_{t=0}^{n-1} x_t y_{t \oplus \tau} \quad \& \tag{5}$$

$$\lambda_c \geq \sum_{t=0}^{n-1} y_t \, x_{t \oplus \tau} \quad (6)$$

Here $t \ominus \tau$ implies $(t + \tau) \bmod n$.

1. The autocorrelation of any code with un-shifted version of itself results in weight w, which is also auto-correlation peak.
2. The autocorrelation of one code with its shifted version can result in overlap of 0 to (w-1) bits. The maximum of this value is called auto correlation constraint is λ_a.
3. The cross correlation of one code with other code and its shifted sequences can result in overlap of 0 to (w - 1) bits . The maximum of this value is called cross correlation constraint λ_c.

4. The maximum number of optical orthogonal codes, Z, is given by the following Johnson bounds. Johnson bound A is for

$$\lambda_a = \lambda_c = \lambda,\ z\,(n, w, \lambda) \leq \left\lfloor \frac{1}{w} \left\lfloor \frac{n-1}{\lambda-1} \cdots\cdots \left\lfloor \frac{n-\lambda}{w-\lambda} \right\rfloor \right\rfloor \right\rfloor. \quad (7)$$

IV. DIFFERENT SCHEMES OF GENERATING OPTICAL ORTHOGONAL CODES

(a) One of these methods of generating OOC is based on prime sequences [1]. In this a set of optical orthogonal codes ($n, w, \lambda_a, \lambda_c$) is generated for any prime number p. For these codes, the weight w = p, length $n = p^2$, autocorrelation constraint $\lambda_a = p - 1$ and cross correlation constraint $\lambda_c = 2$. The number of optical orthogonal codes in this set are given by $N = p$.

(b) In another type of OOC based on "Quasi prime" [2] code set ($n, w, \lambda_a, \lambda_c$), $n = q\,p$, where $(r - 1)\, p < q < r\, p$. Here p is a prime number , q and r are positive integers: weight $w = q$, autocorrelation constraint $\lambda_a = (p - 1)r$,

Cross correlation constraint $\lambda_c = 2r$

and the number of code-words N= p.

(c) In OOCs based on Quadratic Congruences [3], the orthogonal set ($n, w, \lambda_a, \lambda_c$) is constructed for the length $n = p^2$,weight $w = p$, $\lambda_a = 2$, $\lambda_c = 4$. When the length of the code is extended, we get the extended quadratic congruence code. It can be used for construction of codes of length $n = p\,(2p-1)$, weight $w = p$, autocorrelation constraint $\lambda_a = 1$, and cross correlation constraint $\lambda_c = 2$.

(d) In projective Geometry based OOCs [4], a Projective geometry PG (m, q) of order m is constructed from a vector space V (m+1 , q) of dimension m + 1 over GF (q) . Where $GF(q)$ is Galois field with q elements. An s-space in a PG (m, q) corresponds to an (s+1) dimensional space through the origin in V (m+1,q). Here one dimensional subspaces of V are the points and the two dimensional subspaces of V are the lines.

Number of points n in PG (m, q), $n = [(q^{m+1} - 1)/(q - 1)]$ will give the length of the codeword.

Number of points in the (s - 1)–space are $\lambda = (q^{(s+1)} - 1)/(q-1)$ in the s-space, will give the weight of the codeword.

The intersection of two s-space is an (s-1) space. Number of points in the (s - 1)-space are $\lambda = ((q^s - 1)/(q - 1))$ = max (λ_a , λ_c). The cyclic shift of an s-space is also an s-space. The orbit is the set of all s-spaces that are cyclic shift of each other. The number of code words is always equal to number of complete orbits. A codeword consists of discrete logarithm of points in each representative s-space.

Total number of s-spaces is $M_s = \frac{\binom{n}{s+1}}{\binom{w}{s+1}}$. Total number of code words constructed using PG (m, q) for given value of s are equal to $M = \left\lfloor \frac{M_s}{n} \right\rfloor$.

(e) In OOC's based on error Correcting codes [5] ,a 't' error correcting code is represented by (n, d, w),where n is length, d is minimum Hamming distance between any two code-words ,w is the constant weight of a code from the code set .

The minimum distance $d \geq 2t + 1$. An OOC ($n, w, \lambda_a, \lambda_c$) is equivalent to constant weight error correcting codes with minimum distance $d = 2w-2\lambda$, where λ is maximum of (λ_a, λ_c). Only those error correcting codes are selected for optical orthogonal code set whose cyclic shifted versions are also code word.

(f) In optical orthogonal codes using Hadamard matrix, any Hadamard matrix of order n can be used to generate the matrix of order n - 1 by deleting first row and first column. The rows of the matrix of order n-1 form a code set. From the code set the repeated or cyclically shifted codes are included only once to form the optical orthogonal set of length $n = 4t - 1$; weight = $2t - 1$; $\lambda_a = t - 1$; and $\lambda_c = t$; here t is any positive integer [6].

(g) In asymptotically optimal optical orthogonal codes λ = max (λ_a ,λ_c) , we construct a code (rp^n, r, 1, 1) OOC with size $L(p^{n-1}+p^{n-2}+p^{n-3}+\ldots\ldots+1)$ by using the structure of $Z\,p^n$, the ring of integers modulo p^n, where p is an odd prime with p is an odd prime with $p - 1 = r$. L for positive integers r and L, and n is a positive integer.

(h) Multilevel prime codes are constructed *i.e.* (LXN ,w, $\lambda_a = 0$,$\lambda_c = n$) codes with P^n code matrices from Galois field GF(P) and an arbitrary maximum cross correlation function $\lambda_c = n$, where L is the number of time slots, w is the code weight ,and λ_a is the number of maximum autocorrelation sidelobe.

(i) OOCs based on sequence pair (OOCP): The auto correlation of traditional address codes requires the sender's codeword be same as the receiver's codeword which is used to calculate the autocorrelation function. This limits the existing space of address codes with good correlation properties. In OOC's based on sequence pair, the autocorrelation of sequence pair is expressed as cross correlation of these two sequences. The OOCP's of capacity 1 are obtained by exhaustive algorithm. Finally, OOCP of larger capacity can be obtained by recursive construction method [7].

Two sequences (or codes) $x = (x_0, x_1, x_2,\ldots, x_{n-1})$ and $y = (y_0, y_1, y_2,\ldots, y_{n-1})$ from the set of {0,1} sequence pairs, of length n, code weights w_1,w_2 , are called Optical Orthogonal Code Pairs (OOCP), and denoted as OOCP = (x , y). Also OOCP is indicated by parameters (n, w_1, w_2, e, λ_a, λ_c), where e is the auto correlation in the same phase, λ_a is the autocorrelation in the different phase, and λ_a is the cross-correlation constraints value.

The autocorrelation function of (x, y) is defined by the cross correlation between x and y:

$$R_{auto}(\tau) = \sum_{i=0}^{n-1} x_i\, y_{i\oplus\tau}$$

Here $i\oplus\tau$ represents $(i+\tau)$ mod n, τ represents displacement. If $x = y$ in OOCP become the traditional OOC codes.

(j) Construction of 2D optical Codes using (n, w, 2 ,2) OOCs : These 2D codes provide larger code cardinality than other 2D codes by relaxing the maximum cross-correlation function from one to two [8]. These codes potentially support more subscribers and heavier bursty asynchronous traffic with gradual performance degradation as traffic load increases.

In this method, a new family of (m x n, w, $\lambda_a = 2$, $\lambda_c = 2$) 2D codes is constructed ,where m is the number of available wavelengths, n is the code length ,w is the code weight, λ_a denotes the maximum autocorrelation sidelobe, and λ_c denotes the maximum cross correlation function. For any code matrix X = [$x_{i,j}$] in the 2D codes, the periodic autocorrelation sidelobes are bounded by a positive integer λ_a , such that $\sum_{i=0}^{m-1}\sum_{j=0}^{n-1} x_{i,j}\, x_{i,j\oplus\tau} \le \lambda_a$, where τ Є[1,n-1] , $x_{i,j}$ Є {0,1}is an element of X at the i^{th} row and j^{th} column , and "$\oplus$"denote the modulo n addition. And also for any two distinct code matrices X= [$x_{i,j}$] and Y = [$y_{i,j}$] in the 2D codes ,the periodic cross correlation functions are bounded by a positive integer λ_c ,such that $\sum_{i=0}^{m-1}\sum_{j=0}^{n-1} x_{i,j}\, y_{i,j\oplus\tau} \le \lambda_c$, where τ Є[1, n-1] , $y_{i,j}$ Є {0,1}is an element of Y at the i^{th} row and j^{th} column.

V. CONCLUSION

Several constructions of optical orthogonal codes have been proposed. The two dimensional codes overcome the drawbacks of nonlinear effects in large spread sequences of one dimensional (1-D) unipolar codes in fiber optic code division multiple access networks. The performances of all the codes can be verified by simulation using Matlab. The performances are analyzed by considering only Multiple Access Interference (MAI), which is the major source of noise in the broadcast FO-CDMA systems. The codes having better cardinality and spectral efficiency with minimal correlation values, are most suitable for FO-CDMA networks.

VI. REFERENCES

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