Fully Fuzzy Linear System

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***Abstract*— In this paper we intend to solve the fully fuzzy linear system of the form $\tilde{A} \otimes \tilde{x} = \tilde{b}$ and $\tilde{A}_1 \otimes \tilde{x} = \tilde{A}_2 \otimes \tilde{x} \oplus \tilde{b}$ where $\tilde{A}$, $\tilde{A}_1$ and $\tilde{A}_2$ are m x n fuzzy matrices consisting of positive fuzzy numbers , the unknown vector $\tilde{x}$ is a vector consisting of n positive fuzzy numbers and the constant $\tilde{b}$ are vectors consisting of m positive fuzzy numbers, using Simplex method. In this paper we considered the fuzzy numbers are trapezoidal since triangular fuzzy numbers are particular case of trapezoidal fuzzy numbers.**

***Keywords*— Fully Fuzzy Linear system, Dual Fully Fuzzy Linear System, Trapezoidal numbers, Simplex method.**

I. INTRODUCTION

System of simultaneous linear equations plays a vital role in mathematics, Operations Research, Statistics, Physics, Engineering and Social Sciences etc. In many applications at least some of the system's parameters and measurements are represented by fuzzy numbers rather than crisp numbers. Therefore it is imperative to develop mathematical models and numerical procedures to solve such a fuzzy linear system. The general model of a fuzzy linear system whose coefficient matrix is crisp and the right hand side column is an arbitrary fuzzy vector. In the fully fuzzy linear system all the parameters are considered to be fuzzy numbers. In this paper we considered square and non-square fully fuzzy linear system of the form $\tilde{A} \otimes \tilde{x} = \tilde{b}$ and $\tilde{A}_1 \otimes \tilde{x} = \tilde{A}_2 \otimes \tilde{x} \oplus \tilde{b}$ with trapezoidal fuzzy numbers which are non-negative. Amit Kumar. et.al. [1] discussed consistency of the fully fuzzy linear system and the nature of the solutions. Yosef Jafarzadeh [2] propounds duality of fully fuzzy linear system of the form $\tilde{A} \otimes \tilde{X} = \tilde{B} \otimes \tilde{X} \oplus \tilde{C}$ using direct method. Mosleh et al. [3] suggested solution of fully fuzzy linear systems of the form $\tilde{A}_1 \otimes \tilde{x} = \tilde{A}_2 \otimes \tilde{x} \oplus \tilde{b}$ by ST method. S.H. Neseri. et al. [4] proposed a LU decomposition method for solving fully fuzzy linear system with triangular fuzzy numbers.

The structure of this paper is organized as follows

In Section II, we present some basic concepts of fuzzy set theory and define a fully fuzzy linear system of equations. In Section III, we have given the general model of dual fully fuzzy linear system. In Section IV, Solution of simultaneous linear equations using simplex method is discussed. Section V deals with Numerical examples to illustrate the methods. Section IV ends this paper with conclusion and References.

II. PRELIMINARIES

A. Definition: A fuzzy subset $\tilde{A}$ of R is defined by its membership function $\mu_{\tilde{A}} : R \rightarrow [0,1]$, which assigns a real number $\mu_{\tilde{A}}$ in the interval [0, 1], to each element $x \in R$, Where the value of $\mu_{\tilde{A}}$ at x shows the grade of membership of x in $\tilde{A}$.

B. Definition: A fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \frac{m-x}{\alpha}, & m - \alpha \le x \le m, \alpha > 0 \\ 1, & m < x < n \\ 1 - \frac{x-n}{\beta}, & n \le x \le n + \beta, \beta > 0 \\ 0, & \text{otherwise} \end{cases}$$

C. Definition: A fuzzy number $\tilde{A}$ *is* called positive (negative), denoted by $\tilde{A} > 0$ ($\tilde{A} < 0$), if its membership function $\mu_{\tilde{A}}(x)$ satisfies $\mu_{\tilde{A}}(x) = 0, \forall x \le 0$ ($\forall x \ge 0$).

D. Definition: Two fuzzy numbers $\tilde{A} = (m, n, \alpha, \beta)$ and $\tilde{B} = (p, q, \gamma, \delta)$ are said to be equal if and only if $m = p$, $n = q$, $\alpha = \gamma$, and $\beta = \delta$.

E. Definition: A Trapezoidal fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be zero trapezoidal fuzzy number if and only if $m= 0$, $n =0$, $\alpha =0$, $\beta=0$.

F. Definition: Let $\tilde{A} = (\tilde{a}_{ij})$ and $\tilde{B} = (\tilde{b}_{ij})$ be two m X n and n X p fuzzy matrices. We define $\tilde{A} \otimes \tilde{B} = \tilde{C} = (\tilde{c}_{ij})$ Which is the m X p matrix where

$$\tilde{C}_{ij} = \sum_{k=1,2....n}^{\oplus} \tilde{a}_{ik} \otimes \tilde{b}_{kj}$$

G. Arithmetic operations on trapezoidal numbers

Let $\tilde{A}_1 = (m, n, \alpha, \beta)$ and $\tilde{A}_2 = (p, q, \gamma, \delta)$ be two trapezoidal fuzzy numbers then

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (m, n, \alpha, \beta) \oplus (p, q, \gamma, \delta)$
$= (m+p, n+q, \alpha+\gamma, \beta+\delta)$
(ii) $-\tilde{A}_1 = -(m, n, \alpha, \beta) = (-n, -m, \beta, \alpha)$
(iii) $\tilde{A}_1 \geq 0$ and $\tilde{A}_1 \geq$ o then
$\tilde{A}_1 \otimes \tilde{A}_2 = (m, n, \alpha, \beta) \otimes (p, q, \gamma, \delta)$
$\cong (mp, nq, m\gamma + p\alpha, n\delta + q\beta)$

H. Definition: A matrix $\tilde{A} = (\tilde{a}_{ij})$ is called a fuzzy matrix, if each element of $\tilde{A}$ is a fuzzy number. A fuzzy matrix $\tilde{A}$ will be positive and denoted by $\tilde{A} > 0$, if each element of $\tilde{A}$ be positive. We may represent m x n fuzzy matrix $\tilde{A} = (\tilde{a}_{ij})_{mXn}$ such that $\tilde{a}_{ij} = (a_{ij}, b_{ij}, m_{ij}, n_{ij})$, with the new notation $\tilde{A} = (A, B, M, N)$, where $A = (a_{ij}), B = (b_{ij}), M = (m_{ij}), N = (n_{ij})$ are four m x n crisp matrices.

I. Definition : A square fuzzy matrix $\tilde{A} = (\tilde{a}_{ij})$ will be an upper triangular fuzzy matrix, if $\tilde{a}_{ij} = \tilde{0} = (0,0,0,0)\ \forall$ i > j, and a square fuzzy matrix $\tilde{A} = (\tilde{a}_{ij})$ will be a lower triangular fuzzy matrix, if $\tilde{a}_{ij} = \tilde{0} = (0,0,0,0) \forall$ i < j.

J. Definition: Consider the n x n fuzzy linear system of equations

$$(\tilde{a}_{11} \otimes \tilde{x}_1) \oplus (\tilde{a}_{12} \otimes \tilde{x}_2) \oplus \dots \dots \dots \oplus (\tilde{a}_{1n} \otimes \tilde{x}_n) = \tilde{b}_1$$

$$(\tilde{a}_{21} \otimes \tilde{x}_1) \oplus (\tilde{a}_{22} \otimes \tilde{x}_2) \oplus \dots \dots \dots \oplus (\tilde{a}_{2n} \otimes \tilde{x}_n) = \tilde{b}_2$$

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$$(\tilde{a}_{n1} \otimes \tilde{x}_1) \oplus (\tilde{a}_{n2} \otimes \tilde{x}_2) \oplus \dots \dots \dots \oplus (\tilde{a}_{nn} \otimes \tilde{x}_n) = \tilde{b}_n$$

The matrix form of the above equations is $\tilde{A} \otimes \tilde{x} = \tilde{b}$

Where the coefficient matrix $\tilde{A} = (\tilde{a}_{ij})$, $1 \leq i, j \leq n$ is a n X n fuzzy matrix and $\tilde{x}_j \tilde{b}_j \in F(R)$. This system is called a fully fuzzy linear system.

To find a solution of the fully fuzzy linear system $\tilde{A} \otimes \tilde{x} = \tilde{b}$ where $\tilde{A} = (A, B, M, N)$, $\tilde{b} = (b, g, h, k) \geq 0$ and $\tilde{x} = (x, y, z, w) \geq 0$ We have

$(A, B, M, N) \otimes (x, y, z, w) = (b, g, h, k)$

Using G (iii) we have

$(Ax, By, Az + Mx, Bw + Ny) = (b, g, h, k)$

Using D we have

$Ax = b$
$By = g$
$Az + Mx = h$
$Bw + Ny = k$

III. DUAL FULLY FUZZY LINEAR SYSTEM

A fully fuzzy linear system is of the form $\tilde{A}_1 \otimes \tilde{x} = \tilde{A}_2 \otimes \tilde{x} \oplus \tilde{b}$ is called dual fully fuzzy linear system, where $\tilde{A}_1$ and $\tilde{A}_2$ are m x n fuzzy matrices consisting of positive fuzzy numbers , the unknown vector $\tilde{x}$ is a vector consisting of n positive fuzzy numbers and the constant $\tilde{b}$ are vectors consisting of m positive fuzzy numbers

To find a solution of dual fully fuzzy linear system

$$\tilde{A}_1 \otimes \tilde{x} = \tilde{A}_2 \otimes \tilde{x} \oplus \tilde{b}$$

Where $\tilde{A}_1 = (A_1, B_1, M_1, N_1)$, $\tilde{A}_2 = (A_2, B_2, M_2, N_2)$, $\tilde{b} = (b, g, h, k)$ and $\tilde{x} = (x, y, z, w)$

$\tilde{A}_1, \tilde{A}_2, \tilde{b}$ and $\tilde{x} \geq 0$

$(A_1, B_1, M_1, N_1) \otimes (x, y, z, w) =$
$(A_2, B_2, M_2, N_2) \otimes (x, y, z, w) \oplus (b, g, h, k)$

Using II G (iii) we get

$(A_1x, B_1y, A_1z + M_1x, B_1w + N_1y) = (A_2x, B_2y, A_2z + M_2x, B_2w + N_2y) + (b, g, h, k)$

Using II G (i) we get

$(A_1x, B_1y, A_1z + M_1x, B_1w + N_1y) = (A_2x + b, B_2y + g, A_2z + M_2x + h, B_2w + N_2y + k)$

Using II D we get
$A_1x = A_2x + b$
$\Rightarrow (A_1 - A_2)x = b$

$B_1y = B_2y + g$
$\Rightarrow (B_1 - B_2)y = g$

$A_1z + M_1x = A_2z + M_2x + h$
$\Rightarrow (A_1 - A_2)z = h - (M_1 - M_2)x$

$B_1w + N_1y = B_2w + N_2y + k$
$\Rightarrow (B_1 - B_2)z = k - (N_1 - N_2)y$

Let us take $(A_1 - A_2) = A$, $(B_1 - B_2) = B$, $(M_1 - M_2) = M$, $(N_1 - N_2) = N$, the above equations becomes

Ax = b

By=g

$Az = h - Mx$

$Bw = k - Ny$

IV. SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS USING SIMPLEX METHOD

Let us consider the system of m simultaneous equations in n unknowns: Ax = b
Where A is an m x n real matrix, $x^T \in R^n$ and b is an m x 1 real matrix.
For the solution of the simultaneous equations we introduce a dummy objective function $z = 0.x - 1.x_a$
Where x_a are artificial variables, and $x = x_r' - x_r''$. Where $x_r' \geq 0,\ x_r'' \geq 0$.
The formulated Linear programming problem is then solved by simplex method. The optimal solution of this linear programming problem.

V. NUMERICAL EXAMPLE

1. Solve the following fully fuzzy linear system

$(3,6,2,2) \otimes (x_1, y_1, z_1, w_1) \oplus (4, 6,1,2) \otimes (x_2, y_2, z_2, w_2) = (27,66,26, 58)$

$(4,5,1,1) \otimes (x_1, y_1, z_1, w_1) \oplus (5, 8,1,2) \otimes (x_2, y_2, z_2, w_2) = (35,70,25,55)$

Solution: The given fully fuzzy linear system can be written as

$$\begin{bmatrix} (3,6,2,2) & (4,6,1,2) \\ (4,5,1,1) & (5,8,1,2) \end{bmatrix} \begin{bmatrix} (x_1, y_1, z_1, w_1) \\ (x_2, y_2, z_2, w_2) \end{bmatrix} = \begin{bmatrix} (27,66,26,58) \\ (35,70,25,55) \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 6 & 6 \\ 5 & 8 \end{bmatrix}, M = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, N = \begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}$$

$$b = \begin{bmatrix} 27 \\ 35 \end{bmatrix}, g = \begin{bmatrix} 66 \\ 70 \end{bmatrix}, h = \begin{bmatrix} 26 \\ 25 \end{bmatrix}, k = \begin{bmatrix} 58 \\ 55 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

Using II J we have

Ax = b
$3x_1 + 4x_2 = 27$
$4x_1 + 5x_2 = 35$ Using IV we have
Let $x_1 = x_1' - x_1''$ and $x_2 = x_2' - x_2''$ Where $x_1' \geq 0$, $x_1'' \geq 0, x_2' \geq 0,\ x_2'' \geq 0$..
The objective function Maximize $\mathbb{Z} = 0.x_1 + 0.x_2 - A_1 - A_2$
Subject to $3x_1' - 3x_1'' + 4x_2' - 4x_2'' + x_3 = 27$
$4x_1' - 4x_1'' + 5x_2' - 5x_2'' + x_4 = 35$
Where $x_1' \geq 0,\ x_1'' \geq 0, x_2' \geq 0,\ x_2'' \geq 0, x_3 \geq 0, x_4 \geq 0$
Solving using iterative simplex table we have $x_1 = 5, x_2 = 3$

By = g
$6y_1 + 6y_2 = 66$
$5y_1 + 8y_2 = 70$ Using IV we have
Let $y_1 = y_1' - y_1''$ and $y_2 = y_2' - y_2''$ Where $y_1' \geq 0,\ y_1'' \geq 0, y_2' \geq 0,\ y_2'' \geq 0$..
The objective function Maximize $\mathbb{Z} = 0.y_1 + 0.y_2 - A_1 - A_2$
Subject to $6y_1' - 6y_1'' + 6y_2' - 6y_2'' + y_3 = 66$
$5y_1' - 5y_1'' + 8y_2' - 8y_2'' + y_4 = 70$
Where $y_1' \geq 0,\ y_1'' \geq 0, y_2' \geq 0,\ y_2'' \geq 0, y_3 \geq 0, y_4 \geq 0$
Solving using iterative simplex table we have $y_1 = 6, y_2 = 5$

Az + Mx = h
$\Rightarrow$ Az = h – Mx
h-Mx = $\begin{bmatrix} 13 \\ 17 \end{bmatrix}$

$3z_1 + 4z_2 = 13$
$4z_1 + 5z_2 = 17$ Using IV we have
Let $z_1 = z_1' - z_1''$ and $z_2 = z_2' - z_2''$ Where $z_1' \geq 0,\ z_1'' \geq 0, z_2' \geq 0,\ z_2'' \geq 0$..
The objective function Maximize $\mathbb{Z} = 0.z_1 + 0.z_2 - A_1 - A_2$
Subject to $3z_1' - 3z_1'' + 4z_2' - 4z_2'' + z_3 = 13$
$4z_1' - 4z_1'' + 5z_2' - 5z_2'' + z_4 = 17$
Where $z_1' \geq 0,\ z_1'' \geq 0, z_2' \geq 0,\ z_2'' \geq 0, z_3 \geq 0, z_4 \geq 0$
Solving using iterative simplex table we have $z_1 = 3, z_2 = 1$

Bw +Ny = k
$\Rightarrow$ Bw = k – Ny

k-Ny = $\begin{bmatrix} 36 \\ 39 \end{bmatrix}$

$6w_1 + 6w_2 = 36$
$5w_1 + 8w_2 = 39$ Using IV we have
Let $w_1 = w_1' - w_1''$ and $w_2 = w_2' - w_2''$ Where $w_1' \geq 0$, $w_1'' \geq 0, w_2' \geq 0,\ w_2'' \geq 0$..
The objective function Maximize $\mathbb{Z} = 0.w_1 + 0.w_2 - A_1 - A_2$
Subject to $6w_1' - 6w_1'' + 6w_2' - 6w_2'' + w_3 = 36$
$5w_1' - 5w_1'' + 8w_2' - 8w_2'' + w_4 = 39$
Where $w_1' \geq 0,\ w_1'' \geq 0, w_2' \geq 0,\ w_2'' \geq 0, w_3 \geq 0, w_4 \geq 0$
Solving using iterative simplex table we have $w_1 = 3, w_2 = 3$

$\tilde{x}_1 = (5,6,3,3)$ and $\tilde{x}_2 = (3,5,1,3)$

2. Solve the following fully fuzzy linear system

$(2,2,1,1) \otimes (x_1, y_1, z_1, w_1) \oplus (8,8,2,2) \otimes (x_2, y_2, z_2, w_2) = (20,30,48,61)$
$(1,2,3,4) \otimes (x_1, y_1, z_1, w_1) \oplus (5,6,7,8) \otimes (x_2, y_2, z_2, w_2) = (11,26,48,86)$
$(1,3,5,7) \otimes (x_1, y_1, z_1, w_1) \oplus (2,4,6,8) \otimes (x_2, y_2, z_2, w_2) = (8,29,50,108)$

Solution: The given fully fuzzy linear system can be written as

$$A = \begin{bmatrix} 2 & 8 \\ 1 & 5 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & 8 \\ 2 & 6 \\ 3 & 4 \end{bmatrix}, M = \begin{bmatrix} 1 & 2 \\ 3 & 7 \\ 5 & 6 \end{bmatrix}, N = \begin{bmatrix} 1 & 2 \\ 4 & 8 \\ 7 & 8 \end{bmatrix}$$

$$b = \begin{bmatrix} 20 \\ 11 \\ 8 \end{bmatrix}, g = \begin{bmatrix} 30 \\ 26 \\ 29 \end{bmatrix}, h = \begin{bmatrix} 48 \\ 48 \\ 50 \end{bmatrix}, k = \begin{bmatrix} 61 \\ 86 \\ 108 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

Using II J we have

Ax = b
$2x_1 + 8x_2 = 20$
$x_1 + 5x_2 = 11$
$x_1 + 2x_2 = 8$ Using IV we have
Let $x_1 = x_1' - x_1''$ and $x_2 = x_2' - x_2''$ Where $x_1' \geq 0,\ x_1'' \geq 0, x_2' \geq 0,\ x_2'' \geq 0$..
The objective function Maximize $\mathbb{Z} = 0.x_1 + 0.x_2 - A_1 - A_2$
Subject to $2x_1' - 2x_1'' + 8x_2' - 8x_2'' + x_3 = 20$
$x_1' - x_1'' + 5x_2' - 5x_2'' + x_4 = 11$
$x_1' - x_1'' + 2x_2' - 2x_2'' + x_5 = 8$
Where $x_1' \geq 0,\ x_1'' \geq 0, x_2' \geq 0,\ x_2'' \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0$
Solving using iterative simplex table we have $x_1 = 6, x_2 = 1$

By = g
$2y_1 + 8y_2 = 30$
$2y_1 + 6y_2 = 26$
$3y_1 + 4y_2 = 29$ Using IV we have
Let $y_1 = y_1' - y_1''$ and $y_2 = y_2' - y_2''$ Where $y_1' \geq 0,\ y_1'' \geq 0, y_2' \geq 0,\ y_2'' \geq 0$..
The objective function Maximize $\mathbb{Z} = 0.y_1 + 0.y_2 - A_1 - A_2$
Subject to $2y_1' - 2y_1'' + 8y_2' - 8y_2'' + y_3 = 30$
$2y_1' - 2y_1'' + 6y_2' - 6y_2'' + y_4 = 26$
$3y_1' - 3y_1'' + 4y_2' - 4y_2'' + y_5 = 29$

Where $y_1' \geq 0,\ y_1'' \geq 0, y_2' \geq 0,\ y_2'' \geq 0, y_3 \geq 0, y_4 \geq 0, y_5 \geq 0$

Solving using iterative simplex table we have $y_1 = 7, y_2 = 2$

Az + Mx = h
⇒ Az = h – Mx

$$\text{h-Mx} = \begin{bmatrix} 40 \\ 23 \\ 14 \end{bmatrix}$$

$2z_1 + 8z_2 = 40$
$z_1 + 5z_2 = 23$
$z_1 + 2z_2 = 14$ Using IV we have
Let $z_1 = z_1' - z_1''$ and $z_2 = z_2' - z_2''$ Where $z_1' \geq 0,\ z_1'' \geq 0, z_2' \geq 0,\ z_2'' \geq 0$..
The objective function Maximize $\mathbb{Z} = 0.z_1 + 0.z_2 - A_1 - A_2$
Subject to $2z_1' - 2z_1'' + 8z_2' - 8z_2'' + z_3 = 40$
$z_1' - z_1'' + 5z_2' - 5z_2'' + z_4 = 23$
$z_1' - z_1'' + 2z_2' - 2z_2'' + z_5 = 14$
Where $z_1' \geq 0,\ z_1'' \geq 0, z_2' \geq 0,\ z_2'' \geq 0, z_3 \geq 0, z_4 \geq 0, z_5 \geq 0$
Solving using iterative simplex table we have $z_1 = 8, z_2 = 3$

Bw +Ny = k
⇒ Bw = k – Ny

$$\text{k-Ny} = \begin{bmatrix} 50 \\ 42 \\ 43 \end{bmatrix}$$

$2y_1 + 8y_2 = 50$
$2y_1 + 6y_2 = 42$
$3y_1 + 4y_2 = 43$ Using IV we have

Let $w_1 = w_1' - w_1''$ and $w_2 = w_2' - w_2''$ Where $w_1' \geq 0, w_1'' \geq 0, w_2' \geq 0,\ w_2'' \geq 0$..
The objective function Maximize $\mathbb{Z} = 0.w_1 + 0.w_2 - A_1 - A_2$
Subject to $2w_1' - 2w_1'' + 8w_2' - 8w_2'' + w_3 = 50$
$2w_1' - 2w_1'' + 6w_2' - 6w_2'' + w_4 = 42$
$3w_1' - 3w_1'' + 4w_2' - 4w_2'' + w_5 = 43$
Where $w_1' \geq 0,\ w_1'' \geq 0, w_2' \geq 0,\ w_2'' \geq 0, w_3 \geq 0, w_4 \geq 0, w_5 \geq 0$
Solving using iterative simplex table we have $w_1 = 9, w_2 = 4$

Using 3 we have $\tilde{x}_1 = (6,7,8,9)$ $\tilde{x}_2 = (1,2,3,4)$

3. Solve the following dual fully fuzzy linear system

$(3,6,2,2) \otimes (\mathrm{x}_1, \mathrm{y}_1, \mathrm{z}_1, \mathrm{w}_1) \oplus (4,6,1,2) \otimes (\mathrm{x}_2, \mathrm{y}_2, \mathrm{z}_2, \mathrm{w}_2) = (4,5,1,1) \otimes (\mathrm{x}_1, \mathrm{y}_1, \mathrm{z}_1, \mathrm{w}_1) \oplus (1,2,3,4) \otimes (\mathrm{x}_2, \mathrm{y}_2, \mathrm{z}_2, \mathrm{w}_2) \oplus (14,26,9,26)$

$(1,3,5,7) \otimes (\mathrm{x}_1, \mathrm{y}_1, \mathrm{z}_1, \mathrm{w}_1) \oplus (2,4,6,8) \otimes (\mathrm{x}_2, \mathrm{y}_2, \mathrm{z}_2, \mathrm{w}_2)$
$= (2,3,4,5) \otimes (\mathrm{x}_1, \mathrm{y}_1, \mathrm{z}_1, \mathrm{w}_1 \oplus (1,3,4,5) \otimes (\mathrm{x}_2, \mathrm{y}_2, \mathrm{z}_2, \mathrm{w}_2) \oplus (4,6,15,30)$

Solution

$A_1 = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$, $A_2 = \begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix}$, $B_1 = \begin{bmatrix} 6 & 6 \\ 3 & 4 \end{bmatrix}$, $B_2 = \begin{bmatrix} 5 & 2 \\ 3 & 3 \end{bmatrix}$,

$M_1 = \begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix}$, $M_2 = \begin{bmatrix} 1 & 3 \\ 4 & 4 \end{bmatrix}$, $N_1 = \begin{bmatrix} 2 & 2 \\ 7 & 8 \end{bmatrix}$, $N_2 = \begin{bmatrix} 1 & 4 \\ 5 & 5 \end{bmatrix}$

$b = \begin{bmatrix} 14 \\ 4 \end{bmatrix}$, $g = \begin{bmatrix} 26 \\ 6 \end{bmatrix}$, $h = \begin{bmatrix} 9 \\ 15 \end{bmatrix}$, $k = \begin{bmatrix} 26 \\ 30 \end{bmatrix}$

$A = A_1 - A_2 = \begin{bmatrix} -1 & 3 \\ -1 & 1 \end{bmatrix}$

$B = B_1 - B_2 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

M = $M_1 - M_2 = \begin{bmatrix} 1 & -2 \\ 1 & 2 \end{bmatrix}$

N = $N_1 - N_2 = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$

Using III we have

Ax = b
$-x_1 + 3x_2 = 14$
$-x_1 + x_2 = 4$ Using IV we have
Let $x_1 = x_1^{'} - x_1^{''}$ and $x_2 = x_2^{'} - x_2^{''}$ Where $x_1^{'} \geq 0$, $x_1^{''} \geq 0, x_2^{'} \geq 0,\ x_2^{''} \geq 0$..
The objective function Maximize $\mathbb{Z} = 0.x_1 + 0.x_2 - A_1 - A_2$
Subject to $-x_1^{'} + x_1^{''} + 3x_2^{'} - 3x_2^{''} + x_3 = 14$
$-x_1^{'} + x_1^{''} + x_2^{'} - x_2^{''} + x_4 = 4$
Where $x_1^{'} \geq 0,\ x_1^{''} \geq 0, x_2^{'} \geq 0,\ x_2^{''} \geq 0, x_3 \geq 0, x_4 \geq 0$
Solving using iterative simplex table we have $x_1 = 1, x_2 = 5$

By = g
$y_1 + 4y_2 = 26$
$0.y_1 + 1y_2 = 6$ Using IV we have
Let and $y_2 = y_2^{'} - y_2^{''}$ Where $y_1^{'} \geq 0,\ y_1^{''} \geq 0, y_2^{'} \geq 0$, $y_2^{''} \geq y_1 = y_1^{'} - y_1^{''} 0$..
The objective function Maximize $\mathbb{Z} = 0.y_1 + 0.y_2 - A_1 - A_2$
Subject to $y_1^{'} - y_1^{''} + 4y_2^{'} - 4y_2^{''} + y_3 = 26$
$0.y_1^{'} - 0y_1^{''} + y_2^{'} - y_2^{''} + y_4 = 6$
Where $y_1^{'} \geq 0,\ y_1^{''} \geq 0, y_2^{'} \geq 0,\ y_2^{''} \geq 0, y_3 \geq 0, y_4 \geq 0$
Solving using iterative simplex table we have $y_1 = 2, y_2 = 6$

Az + Mx = h
⇒ Az = h – Mx

h - Mx = $\begin{bmatrix} 18 \\ 4 \end{bmatrix}$

$-x_1 + 3x_2 = 18$
$-x_1 + x_2 = 4$ Using IV we have
Let $z_1 = z_1^{'} - z_1^{''}$ and $z_2 = z_2^{'} - z_2^{''}$ Where $z_1^{'} \geq 0,\ z_1^{''} \geq 0, z_2^{'} \geq 0,\ z_2^{''} \geq 0$..
The objective function Maximize $\mathbb{Z} = 0.z_1 + 0.z_2 - A_1 - A_2$
Subject to $-z_1^{'} + z_1^{''} + 3z_2^{'} - 3z_2^{''} + z_3 = 18$
$-z_1^{'} + z_1^{''} + z_2^{'} - z_2^{''} + z_4 = 4$
Where $z_1^{'} \geq 0,\ z_1^{''} \geq 0, z_2^{'} \geq 0,\ z_2^{''} \geq 0, z_3 \geq 0, z_4 \geq 0$
Solving using iterative simplex table we have $z_1 = 3, z_2 = 7$

Bw +Ny = k
⇒ Bw = k – Ny
k-Ny = $\begin{bmatrix} 36 \\ 8 \end{bmatrix}$

$w_1 + 4w_2 = 36$
$0.w_1 + w_2 = 8$ Using IV we have
Let $w_1 = w_1^{'} - w_1^{''}$ and $w_2 = w_2^{'} - w_2^{''}$ Where $w_1^{'} \geq 0$, $w_1^{''} \geq 0, w_2^{'} \geq 0,\ w_2^{''} \geq 0$..
The objective function Maximize $\mathbb{Z} = 0.w_1 + 0.w_2 - A_1 - A_2$
Subject to $w_1^{'} - w_1^{''} + 4w_2^{'} - 4w_2^{''} + w_3 = 36$
$0.w_1^{'} - 0.w_1^{''} + w_2^{'} - w_2^{''} + w_4 = 8$
Where $w_1^{'} \geq 0,\ w_1^{''} \geq 0, w_2^{'} \geq 0,\ w_2^{''} \geq 0, w_3 \geq 0, w_4 \geq 0$
Solving using iterative simplex table we have $w_1 = 4, w_2 = 8$

$\tilde{x}_1 = (1,2,3,4), \tilde{x}_2 = (5,6,7,8)$

4. Solve the following dual fully fuzzy linear system

(3,6,2,2) ⊗ (x_1, y_1, z_1, w_1)⊕(4,6,1,2) ⊗ (x_2, y_2, z_2, w_2) = (4,5,1,1) ⊗ (x_1, y_1, z_1, w_1) ⊕ (1,2,3,4) ⊗ (x_2, y_2, z_2, w_2) ⊕ (14,26,9,26)

(1,3,5,7) ⊗ (x_1, y_1, z_1, w_1)⊕(2,4,6,8) ⊗ (x_2, y_2, z_2, w_2) = (2,3,4,5) ⊗ (x_1, y_1, z_1, w_1) ⊕ (1,3,4,5) ⊗ (x_2, y_2, z_2, w_2) ⊕ (4,6,15,30)

(8,8,2,2) ⊗ (x_1, y_1, z_1, w_1)⊕(7,8,9,10) ⊗ (x_2, y_2, z_2, w_2) = (5,6,7,8) ⊗ (x_1, y_1, z_1, w_1) ⊕ (2,2,1,1) ⊗ (x_2, y_2, z_2, w_2) ⊕ (28,40,79,98)

Solution

$A_1 = \begin{bmatrix} 3 & 4 \\ 1 & 2 \\ 8 & 7 \end{bmatrix}$, $A_2 = \begin{bmatrix} 4 & 1 \\ 2 & 1 \\ 5 & 2 \end{bmatrix}$ $B_1 = \begin{bmatrix} 6 & 6 \\ 3 & 4 \\ 8 & 8 \end{bmatrix}$, $B_2 = \begin{bmatrix} 5 & 2 \\ 3 & 3 \\ 6 & 2 \end{bmatrix}$

$M_1 = \begin{bmatrix} 2 & 1 \\ 5 & 6 \\ 2 & 9 \end{bmatrix}$, $M_2 = \begin{bmatrix} 1 & 3 \\ 4 & 4 \\ 7 & 1 \end{bmatrix}$, $N_1 = \begin{bmatrix} 2 & 2 \\ 7 & 8 \\ 2 & 10 \end{bmatrix}$, $N_2 = \begin{bmatrix} 1 & 4 \\ 5 & 5 \\ 8 & 1 \end{bmatrix}$

$b = \begin{bmatrix} 14 \\ 4 \\ 28 \end{bmatrix}$ $g = \begin{bmatrix} 26 \\ 6 \\ 40 \end{bmatrix}$, $h = \begin{bmatrix} 9 \\ 15 \\ 79 \end{bmatrix}$, $k = \begin{bmatrix} 26 \\ 30 \\ 98 \end{bmatrix}$

$A = A_1 - A_2 = \begin{bmatrix} -1 & 3 \\ -1 & 1 \\ 3 & 5 \end{bmatrix}$

$B = B_1 - B_2 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \\ 2 & 6 \end{bmatrix}$

$M = M_1 - M_2 = \begin{bmatrix} 1 & -2 \\ 1 & 2 \\ -5 & 8 \end{bmatrix}$

$N = N_1 - N_2 = \begin{bmatrix} 1 & -2 \\ 2 & 3 \\ -6 & 9 \end{bmatrix}$

Using III we have
Ax = b
$-x_1 + 3x_2 = 14$
$-x_1 + x_2 = 4$
$3_1 + 5x_2 = 28$ Using IV we have
Let $x_1 = x_1' - x_1''$ and $x_2 = x_2' - x_2''$ Where $x_1' \geq 0,\ x_1'' \geq 0, x_2' \geq 0,\ x_2'' \geq 0$..
The objective function Maximize $\mathbb{Z} = 0.x_1 + 0.x_2 - A_1 - A_2$
Subject to $-x_1' + x_1'' + 3x_2' - 3x_2'' + x_3 = 14$
$-x_1' + x_1'' + x_2' - x_2'' + x_4 = 4$
$3x_1' - 3x_1'' + 5x_2' - 5x_2'' + x_5 = 28$
Where $x_1' \geq 0,\ x_1'' \geq 0, x_2' \geq 0,\ x_2'' \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0$
Solving using iterative simplex table we have $x_1 = 1, x_2 = 5$

By = g
$y_1 + 4y_2 = 26$
$0.y_1 + y_2 = 6$
$2y_1 + 6y_2 = 40$ Using IV we have
Let $y_1 = y_1' - y_1''$ and $y_2 = y_2' - y_2''$ Where $y_1' \geq 0,\ y_1'' \geq 0, y_2' \geq 0,\ y_2'' \geq 0$..
The objective function Maximize $\mathbb{Z} = 0.y_1 + 0.y_2 - A_1 - A_2$
Subject to $y_1' - y_1'' + 4y_2' - 4y_2'' + y_3 = 26$
$0.y_1' - 0.y_1'' + y_2' - y_2'' + y_4 = 6$
$2y_1' - 2y_1'' + 6y_2' - 6y_2'' + y_5 = 40$
Where $y_1' \geq 0,\ y_1'' \geq 0, y_2' \geq 0,\ y_2'' \geq 0, y_3 \geq 0\ , y_4 \geq 0, y_5 \geq 0$
Solving using iterative simplex table we have $y_1 = 2, y_2 = 6$

Az + Mx = h
$\Rightarrow$ Az = h – Mx

h-Mx = $\begin{bmatrix} 18 \\ 4 \\ 44 \end{bmatrix}$

$-z_1 + 3z_2 = 18$
$-z_1 + z_2 = 4$
$3z_1 + 5z_2 = 44$ Using IV we have
Let $z_1 = z_1' - z_1''$ and $z_2 = z_2' - z_2''$ Where $z_1' \geq 0,\ z_1'' \geq 0, z_2' \geq 0,\ z_2'' \geq 0$..
The objective function Maximize $\mathbb{Z} = 0.z_1 + 0.z_2 - A_1 - A_2$
Subject to $-z_1' + z_1'' + 3z_2' - 3z_2'' + z_3 = 18$
$-z_1' + z_1'' + z_2' - z_2'' + z_4 = 4$
$3z_1' - 3z_1'' + 5z_2' - 5z_2'' + z_5 = 44$
Where $z_1' \geq 0,\ z_1'' \geq 0, z_2' \geq 0,\ z_2'' \geq 0, z_3 \geq 0\ ,\ z_4 \geq 0, z_5 \geq 0$
Solving using iterative simplex table we have $z_1 = 3, z_2 = 7$

Bw +Ny = k
$\Rightarrow$ Bw = k – Ny
k-Ny = $\begin{bmatrix} 36 \\ 8 \\ 56 \end{bmatrix}$

$y_1 + 4y_2 = 36$
$0.y_1 + y_2 = 8$
$2y_1 + 6y_2 = 56$ Using IV we have

Let $w_1 = w_1' - w_1''$ and $w_2 = w_2' - w_2''$ Where $w_1' \geq 0, w_1'' \geq 0, w_2' \geq 0,\ w_2'' \geq 0$..
The objective function Maximize $\mathbb{Z} = 0.w_1 + 0.w_2 - A_1 - A_2$
Subject to $w_1' - w_1'' + 4w_2' - 4w_2'' + w_3 = 36$
$0.w_1' - 0.w_1'' + w_2' - w_2'' + w_4 = 8$
$2w_1' - 2w_1'' + 6w_2' - 6w_2'' + w_5 = 56$
Where $w_1' \geq 0,\ w_1'' \geq 0, w_2' \geq 0,\ w_2'' \geq 0, w_3 \geq 0\ ,\ w_4 \geq 0, w_5 \geq 0$
Solving using iterative simplex table we have $w_1 = 4, w_2 = 8$

$\tilde{x}_1 = (1,2,3,4)$ $\tilde{x}_2 = (5,6,7,8)$

VI. CONCLUSION

In this paper, we proposed a new method for solving fully fuzzy linear system $\tilde{A} \otimes \tilde{x} = \tilde{b}$ and dual fully fuzzy linear system $\tilde{A}_1 \otimes \tilde{x} = \tilde{A}_2 \otimes \tilde{x} \oplus \tilde{b}$. We solved 2 x 2, 3 x 2 fully fuzzy linear system and dual fully fuzzy linear system using Simplex method in the form of trapezoidal fuzzy number matrices.

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