Optimization of a Single-Vendor Single-Buyer Fuzzy Inventory Model with Discrete Delivery Order, Random Machine Unavailability Time and Lost Sales

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Abstract - Supply chain management is concerned with the coordination of material and information flows in multi-stage production systems. Inventory management has long been treated as an isolated function solely focused on individual entities, taking into account concerned with single vendor-single buyer and single-vendor multiple buyer models. Integrated single-vendor single-buyer inventory model with multiple deliveries has proved to result in less inventory cost. However, many researchers assumed that the production run is perfect and there is no production delay. In reality, production delay is prevalent due to random machine unavailability and shortages. This study considers lost sales, and two kinds of machine unavailability distributions - uniformly and exponentially distributed. A classical optimization technique is used to device an optimal solution and a numerical example is provided to illustrate the theory. In this model we consider the fuzzy total cost under crisp order quantity or fuzzy order quantity in order to extend the traditional inventory model to the fuzzy environment. We use Function Principle as arithmetic operators of fuzzy total cost and use the Graded Mean Integration Representation Method to defuzzify the fuzzy total cost. Then we use the Kuhn-Tucker Method to find the optimal order quantity of the fuzzy order inventory model. The results show that delivery frequency has significant effect on the optimal total cost and a higher lost sales cost will result in a higher delivery frequency.

I. INTRODUCTION

Due to unreliable production system, vendors may not deliver products to the buyers when needed, resulting in lost sales. However excessive supplies to fulfill customer's requirement results in higher inventory cost. The inventory cost is one of the dominant costs for many industries. Industries should plan their strategy to provide products and services to the customers at a minimum cost. The order quantity and the time to order are critical decisions for both the manufacturing and the service industries. Some industries implement Just In Time (JIT) System to reduce their inventory cost. In order to support an efficient JIT system, it is important to ensure the reliability of the vendor's production system.

But in reality, there are possibilities that the production process is delayed due to machine unavailability and shortages of materials and facilities. Abboud et al. (2000) developed EPQ models by considering random machine unavailability with backorders and lost sales. the models were extended by Jaber and Abboud (2001) who assumed learning and forgetting in production. later Chung et al (2011) by considering deteriorating items In this study, we assume a JIT system where the buyer who pays the transportation cost, decides the order quantity size of items and requests items delivery in multiple shipments. The vendor products the items using an Economic Production Quantity (EPQ) model. Ideally, the machine starts a production run when the inventory level is equal to zero. In some periods, there is a possibility that the machine may not be available. If this situation occurs, the vendor cannot deliver the predetermined quantity ordered by the buyer, resulting in the buyer's lost sales. We consider two distribution models for the random machine unavailability case. The distribution models represent two different types of distribution. Uniformly distributed means constant number of machine unavailability over a period of time while exponentially distributed means machine unavailability may increase with time. Both cases can occur in real life. Similar distribution types were used by Abboud et al. (2000) and Giri and Dohi (2005).

In section 2, the methodology is introduced. In section 3, discuss with fuzzy EOQ inventory models with different situation. A numerical example is shown in section 4 and section 5 concludes.

II. METHODOLOGY

2.1. Graded Mean Integration Representation Method

Chen and Hsieh introduced Graded mean Integration Representation Method based on the integral value of graded mean h-level of generalized fuzzy number for defuzzifying generalized fuzzy number. Here, we fist define generalized fuzzy number as follows :

Suppose \tilde{A} is a generalized fuzzy number as shown in Fig.1. It is described as any fuzzy subset of the real line R, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions.

- 1. $\mu_{\tilde{\lambda}}(\mathbf{x})$ is a continuous mapping from R to [0, 1],
- $2. \quad \mu_{\tilde{A}}\left(x\right) = 0, \, \text{-}\infty < x \leq a_1,$
- 3. $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$,
- $4. \quad \mu_{\tilde{A}}\left(x\right) = W_A, \, a_2 \leq x \leq a_3,$

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5. $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$,

6.
$$\mu_{\tilde{a}}(\mathbf{x}) = 0, a_4 \le \mathbf{x} < \infty$$

where $0 < W_A \le 1$ and a_1 , a_2 , a_3 and a_4 are real numbers.

This type of generalized fuzzy numbers are denoted as $\tilde{A} = (a_1, a_2, a_3, a_4; \omega_A)_{LR}$ when $\omega_A = 1$, it can be formed as $\tilde{A} = (a_1, a_2, a_3, a_4; \omega_A)_{LR}$. Second, by Graded Mean Integration Representation Method, L^{-1} and R^{-1} are the inverse functions of L and R respectively and the graded mean h-level value of generalized fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4; \omega_A)_{LR}$ is given by $\frac{h}{2}(L^1(h) + R^{-1}(h))$. (see fig.2). Then the graded Mean Integration Representation of P(\tilde{A}) with grade w_A , where

$$P(\tilde{A}) = \frac{\int_{0}^{\omega_{A}} \frac{h}{2} (L^{-1}(h) + R^{-1}(h)) dh}{\int_{0}^{\omega_{A}} h dh} \dots (1)$$

where $0 < h \le w_A$ and $0 < w_A \le 1$.

Throughout this paper, we only use popular trapezoidal fuzzy number as the type of all fuzzy parameters in our proposed fuzzy production inventory models. Let \tilde{B} be a trapezoidal fuzzy number and be denoted as $\tilde{B} = (b_1, b_2, b_3, b_4)$. Then we can get the Graded Mean Integration Representation of \tilde{B} by the formula (1) as

$$P(\tilde{B}) = \frac{\int_{0}^{1} \frac{h}{2} \left[(b_{1}+b_{4}) + h(b_{2}-b_{1}-b_{4}+b_{3}) \right] dh}{\int_{0}^{1} h dh}$$

$$= \frac{b_{1}+2b_{2}+2b_{3}+b_{4}}{6} \qquad \dots (2)$$

$$W_{A} \xrightarrow{\tilde{A}}$$

$$W_{A} \xrightarrow{\tilde{A}}$$

$$H \xrightarrow{\tilde{A$$



2.2. The Fuzzy Arithmetical Operations under Function Principle

Function Principle is introduced by Chen (1985) to treat the fuzzy arithmetical operations by trapezoidal fuzzy numbers. We will use this principle as the operation of addition, multiplication, subtraction and division of trapezoidal fuzzy numbers, because (1) the Function Principle is easier to

calculate than the Extension Principle, 92) the Function Principle will not change the shape of trapezoidal fuzzy number after the multiplication of two trapezoidal fuzzy numbers, but the multiplication of two trapezoidal fuzzy numbers will become drum-like shape fuzzy number by using the Extension Principle (3) if we have to multiply more than four trapezoidal fuzzy numbers then the Extension Principle cannot solve the operation, but the function principle can easily find the result by pointwise computation. Here we describe some fuzzy arithmetical operations under the Function Principle as follows.

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers. Then

(1) The addition of \tilde{A} and \tilde{B} is

 $\widetilde{\mathbf{A}} \oplus \widetilde{\mathbf{B}} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

where a_1 , a_2 , a_3 , a_4 , b_1 , b_2 , b_3 and b_4 are any real numbers.

(2) The multiplication of \widetilde{A} and \widetilde{B} is $\widetilde{B} = (C_1, C_2, C_3, C_4)$ where $T = \{a_1b_1, a_2b_2, a_3b_3, a_4b_4\}$ $T_1 = \{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$ $C_1 = \min T_1, C_2 = \min T_1, C_3 = \max T_1, C_4 = \max T_1$

 $C_1 = \min T_1, C_2 = \min T_1, C_3 = \max T_1, C_4 = \max T_1$ If $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are all zero positive real numbers then

A
$$\otimes$$
 B = (a₁b₁, a₂b₂, a₃b₃, a₄b₄)

(3) $-\tilde{B} = (-b_4, -b_3, -b_2, -b_1)$ then the subtraction of \tilde{A} and \tilde{B} is

 $A \ominus B = \{a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1\}$ where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are any real numbers.

(4)
$$\frac{1}{\tilde{B}} = \tilde{B}^{-1} \left(\frac{1}{b_4}, \frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1} \right)$$
 where b_1, b_2, b_3, b_4

are all positive real numbers. If a_1 , a_2 , a_3 , a_4 , b_1 , b_2 , b_3 and b_4 are all nonzero positive real numbers then the division \tilde{A} and \tilde{B} is

$$\tilde{\mathbf{A}} \oslash \widetilde{\mathbf{B}} = \left(\frac{\mathbf{a}_1}{\mathbf{b}_4}, \frac{\mathbf{a}_2}{\mathbf{b}_3}, \frac{\mathbf{a}_3}{\mathbf{b}_2}, \frac{\mathbf{a}_4}{\mathbf{b}_1}\right)$$

(5) Let
$$\alpha \in \mathbb{R}$$
, then

(i) $\alpha \ge 0, \alpha \otimes \tilde{A} = (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4)$

(ii)
$$\alpha \ge 0, \alpha \otimes A = (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1)$$

2.3. The Kuhn-Tucker Conditions

Taha (1997) discussed how to solve the optimum solution of nonlinear programming problem subject to inequality constraints by using the Kuhn–Tucker conditions. The development of the Kuhn–Tucker conditions is based on the Lagrangean method.

Suppose that the problem is given by

Minimize y = f(x)

Subject to $g_i(x) \ge 0$, $i = 1, 2, \ldots, m$.

The nonnegativity constraints $x \ge 0$, if any, are included in the m constraints.

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The inequality constraints may be converted into equations by using nonnegative surplus variables. Let S_i^2 be the surplus quantity added to the ith constraint $g_i(x) \ge 0$.

Let
$$\lambda = (\lambda_1, \lambda_2, \dots, \lambda_m)$$
, $g(x) = (g_1(x), g_2(x), \dots, g_m(x)) \dots$
and $S_2 = (S_1^2, S_2^2, \dots, S_m^2)$.

The Kuhn–Tucker conditions need x and λ to be a stationary point of the minimization problem, which can be summarized as follows :

$$\begin{cases} \lambda \ \leq \ 0, \\ \nabla f(x) \ - \ \lambda \nabla g(x) \ = \ 0, \\ \lambda_i g_i(x) \ = \ 0, \ i \ = \ 1, \ 2, \ \ldots, \ m. \\ g_i(x) \ \geq \ 0, \ i \ = \ 1, \ 2, \ \ldots, \ m. \end{cases}$$

III. THE EOQ INVENTORY MODEL WITH DISCRETE DELIVER ORDER, RANDOM MACHINE UNAVAILABILITY TIME AND LOST SALES

In this section, we develop a sequential optimization method using Kuhn-Tucker method. We use this method to find the Optimal Economic Order Quantity (EOQ) of the fuzzy inventory model.

3.1. Model Formation

Assume that the unavailability time t is a random variable uniformly distributed over the interval [0, 6]. The probability density function f(t) is given as

$$f(t) = \begin{cases} \frac{1}{b}, & 0 \le t \le b\\ 0, & \text{Otherwise} \end{cases}$$

The vendor and buyer total cost when the production down time is bigger than the upper bound of the machine unavailability time is

$$TUC_{NL} = TVUC_{NL} + TBUC_{NL}$$
$$TUC_{NL} = \frac{A_v D}{qK} \frac{\left[h_v q^2 K \left\{K(1 - (D/p) + 1\right\}\right] D}{2DqK} + \frac{AD}{qK} + \frac{C_v KD}{qK} + \frac{hq^2 KD}{2DqK}$$
$$TUC_{NL} = \frac{A_v D}{qK} \frac{\left[h_v q \left\{K(1 - (D/p) + 1\right\}\right]}{2} + \frac{AD}{qK} + \frac{C_v D}{q} + \frac{hq}{2}$$

...(1)

Taking the derivative of (1) with respect to q and set the value equal to zero we have

$$q_{NL}^{*} = \sqrt{\frac{2(A_{v} + C_{t}K + A)D}{K\left[h_{v}\left\{K(1 - (D/p) + 1\right\} + h\right]}}$$

Throughout this, we use the following variables in order to simplify the treatment of the fuzzy inventory models \tilde{A}_v , \tilde{C}_t , \tilde{A} , \tilde{D} , \tilde{h}_v , \tilde{P} , \tilde{h} are fuzzy parameters.

This fuzzy total cost of the system is

$$\begin{split} & \text{TUC}_{\text{NL}} \\ = \left\{ \begin{aligned} & \frac{A_{v_1} D_1}{qK} \frac{\left[h_{v_1} q \left\{ K(1 - \left(\frac{D_1}{p_4} \right) + 1 \right\} \right]}{2} + \frac{A_1 D_1}{qK} + \frac{C_{v_1} D_1}{q} + \frac{h_1 q}{2}, \\ & \frac{A_{v_2} D_2}{qK} \frac{\left[h_{v_2} q \left\{ K(1 - \left(\frac{D_2}{p_3} \right) + 1 \right\} \right]}{2} + \frac{A_2 D_2}{qK} + \frac{C_{v_2} D_2}{q} + \frac{h_2 q}{2}, \\ & \frac{A_{v_3} D_3}{qK} \frac{\left[h_{v_3} q \left\{ K(1 - \left(\frac{D_3}{p_2} \right) + 1 \right\} \right]}{2} + \frac{A_3 D_3}{qK} + \frac{C_{v_3} D_3}{q} + \frac{h_3 q}{2}, \\ & \frac{A_{v_4} D_4}{qK} \frac{\left[h_{v_4} q \left\{ K(1 - \left(\frac{D_4}{p_1} \right) + 1 \right\} \right]}{2} + \frac{A_4 D_4}{qK} + \frac{C_{v_4} D_4}{q} + \frac{h_4 q}{2} \right\} \end{split}$$

which implies

$$\begin{split} T\widetilde{U}C_{NL} &= ((A_v \otimes D) \oslash (q \otimes K)) + ((h_v \otimes q)/2) \otimes (K \oslash (1 - (D \oslash P)) \oplus 1)) \oplus ((A \otimes D) \oslash (q \otimes K) \\ & \oplus ((C_v \otimes D) \oslash q) + (h \otimes q)/2) \end{split}$$

where \bigoplus , \ominus , \bigotimes and \bigotimes are the fuzzy arithmetical operators under function principle. Suppose

$$\begin{split} \tilde{A}_{v} &= (A_{v_{1}}, A_{v_{2}}, A_{v_{3}}, A_{v_{4}}) \\ \tilde{D} &= (D_{1}, D_{2}, D_{3}, D_{4}) \\ \tilde{h}_{v} &= (h_{v_{1}}, h_{v_{2}}, h_{v_{3}}, h_{v_{4}}) \\ \tilde{P} &= (P_{1}, P_{2}, P_{3}, P_{4}) \\ \tilde{C}_{t} &= (C_{t_{1}}, C_{t_{2}}, C_{t_{3}}, C_{t_{4}}) \\ \tilde{h} &= (h_{1}, h_{2}, h_{3}, h_{4}) \\ \tilde{A} &= (A_{1}, A_{2}, A_{3}, A_{4}) \end{split}$$

are non negative trapezoidal fuzzy numbers. Then we solve the optimal order quantity as the following steps. Second, we defuzzify the fuzzy total inventory cost, using the Graded Mean Integration Representation Method.

The result is $P[TUC_{NI}]$

$$= \frac{1}{6} \left\{ \frac{A_{v_1} D_1}{qK} \frac{\left[h_{v_1} q \left\{K(1 - \left(\frac{D_1}{P_4}\right) + 1\right\}\right]}{2} + \frac{A_1 D_1}{qK} + \frac{C_{t_1} D_1}{q} + \frac{h_1 q}{2} + \frac{A_{t_2} D_2}{qK} + \frac{A_{t_2} D_2}{qK} \frac{\left[h_{v_2} q \left\{K(1 - \left(\frac{D_2}{P_3}\right) + 1\right\}\right]}{2} + \frac{A_2 D_2}{qK} + \frac{C_{t_2} D_2}{q} + \frac{h_2 q}{2} + \frac{h_2 q}{2} + \frac{A_2 D_2}{qK} + \frac{C_{t_2} D_2}{qK} + \frac{A_2 D_2}{qK} + \frac{C_{t_2} D_2}{qK} + \frac{A_2 D_2}$$



Third we can get the optimal order quantity q_{NL}^* , when $P[T\widetilde{U}C_{NL}]$ is minimization. In order to find the minimization of $P[T\widetilde{U}C_{NL}]$ the derivative of $P[T\widetilde{U}C_{NL}]$ with q^* is

 $\frac{\partial P\left[T\tilde{U}C_{NL}\right]}{\partial q} = 0.$

Hence we find the optimal order quantity

$$\tilde{q}^{*} = \begin{cases} \left| \left\{ A_{v_{1}}D_{1} + 2A_{v_{2}}D_{2} + 2A_{v_{3}}D_{3} + A_{v_{4}}D_{4} \right\} \\ + \left\{ A_{1}D_{1} + 2A_{2}D_{2} + 2A_{3}D_{3} + A_{4}D_{4} \right\} \\ + \left\{ C_{t_{1}}KD_{1} + 2C_{t_{2}}KD_{2} + 2C_{t_{3}}KD_{3} + C_{t_{4}}KD_{4} \right\} \right] 2 \\ K \left[\left\{ h_{v_{1}} \left[K \left(1 - \frac{D_{1}}{P_{4}} \right) + 1 \right] + 2h_{v_{2}} \left[K \left(1 - \frac{D_{2}}{P_{3}} \right) + 1 \right] \right] \\ + 2h_{v_{3}} \left[K \left(1 - \frac{D_{3}}{P_{2}} \right) + 1 \right] + h_{v_{4}} \left[K \left(1 - \frac{D_{4}}{P_{1}} \right) + 1 \right] \right\} \\ + \left\{ h_{1} + 2h_{2} + 2h_{3} + h_{4} \right\} \right] \end{cases}$$

3.2. Fuzzy Inventory EOQ Model with Fuzzy Order Quantity

In this section, we introduce the fuzzy inventory EOQ models by changing the crisp order quantity \tilde{q} be a trapezoidal fuzzy number $\tilde{q} = (q_1, q_2, q_3, q_4)$ with $0 \le q_1 \le q_2 \le q_3 \le q_4$. Then we get the fuzzy total cost function as $T\widetilde{UC}_{NL}$

$$= \left\{ \left(\frac{A_{v_1}D_1}{q_4K} \frac{\left[h_{v_1}q_1\left\{K(1-\left(\frac{D_1}{P_4}\right)+1\right\}\right]}{2} + \frac{A_1D_1}{q_4K} + \frac{C_{t_1}D_1}{q_4} + \frac{h_1q_1}{2}\right], \\ \left(\frac{2A_{v_2}D_2}{q_3K} \frac{\left[2h_{v_2}q_2\left\{K(1-\left(\frac{D_2}{P_3}\right)+1\right\}\right]}{2} + \frac{2A_2D_2}{q_3K} + \frac{2C_{t_2}D_2}{q_3} + \frac{h_2q_2}{2}\right) \right\}$$

$$\left\{ \frac{2A_{v_3}D_3}{q_2K} \frac{\left[2h_{v_3}q_3\left\{K(1-\left(\frac{D_3}{P_2}\right)+1\right\}\right]}{2} + \frac{2A_3D_3}{q_2K} + \frac{2C_{v_3}D_3}{q_2} + \frac{h_3q_3}{2}\right\}}{\left(\frac{A_{v_4}D_4}{q_1K} \frac{\left[h_{v_4}q_4\left\{K(1-\left(\frac{D_4}{P_1}\right)+1\right\}\right]}{2} + \frac{A_4D_4}{q_1K} + \frac{C_{t_4}D_4}{q_1} + \frac{h_4q_4}{2}\right)\right\}}$$

Secondly we defuzzify the fuzzy total cost function using the Graded Mean Integration Representation Method with $0 < q_1 \le q_2 \le q_3 \le q_4$.

It will not change the meaning of formula (4.3) if we replace inequality conditions with $0 \le q_1 \le q_2 \le q_3 \le q_4$ into the following inequality constraints $q_2 - q_1 \ge 0$, $q_3 - q_2 \ge 0$, $q_4 - q_3 \ge 0$, $q_1 > 0$.

Thirdly the Kuhn-Tucker condition is used to find the solutions of q_1 , q_2 , q_3 , q_4 to minimize P[TUC_{NL}], subject to $q_2 - q_1 \ge 0$, $q_3 - q_2 \ge 0$, $q_4 - q_3 \ge 0$ and $q_1 > 0$. The Kuhn-Tucker conditions are $\lambda_i \le 0$,

$$\begin{split} \nabla f(P[\widetilde{TUC}_{NL}]) & -\lambda \nabla(q_i) = 0 \\ \lambda_i g_i(q_i) = 0 \\ g_i(q_i) = 0 \end{split}$$

 \Rightarrow

These conditions simplify to the following λ_1 , λ_2 , λ_3 , $\lambda_4 \leq = 0$ & $\nabla f(\widetilde{P}(TUC_{NL}(q_i)) - \lambda \nabla g(q_i) = 0$

$$\begin{split} & \frac{1}{6} \Biggl\{ \Biggl(\frac{A_{v_1} D_1}{q_4 K} \frac{\left[h_{v_1} q_1 \left\{ K(1 - \left(\frac{D_1}{P_4} \right) + 1 \right\} \right]}{2} + \frac{A_1 D_1}{q_4 K} + \frac{C_{t_1} D_1}{q_4} + \frac{h_1 q_1}{2} \Biggr) \\ & + 2 \Biggl(\frac{A_{v_2} D_2}{q_3 K} \frac{\left[h_{v_2} q_2 \left\{ K(1 - \left(\frac{D_2}{P_3} \right) + 1 \right\} \right]}{2} + \frac{A_2 D_2}{q_3 K} + \frac{C_{t_2} D_2}{q_3} + \frac{h_2 q_2}{2} \Biggr) \\ & + 2 \Biggl(\frac{A_{v_3} D_3}{q_2 K} \frac{\left[h_{v_3} q_3 \left\{ K(1 - \left(\frac{D_3}{P_2} \right) + 1 \right\} \right]}{2} + \frac{A_3 D_3}{q_2 K} + \frac{C_{t_3} D_3}{q_2} + \frac{h_3 q_3}{2} \Biggr) \Biggr) \\ & + \Biggl(\frac{A_{v_4} D_4}{q_1 K} \frac{\left[h_{v_4} q_4 \left\{ K(1 - \left(\frac{D_4}{P_1} \right) + 1 \right\} \right]}{2} + \frac{A_4 D_4}{q_1 K} + \frac{C_{t_4} D_4}{q_1} + \frac{h_4 q_4}{2} \Biggr) \Biggr\} \end{split}$$

$$-\lambda_1(q_2-q_1) - \lambda_2(q_3-q_2) - \lambda_3(q_4-q_3) - \lambda_4q_1 = 0$$

which implies

$$\frac{1}{6} \left\{ \frac{h_1}{2} + \frac{h_{v_1} \left[K(1 - \left(\frac{D_1}{P_4}\right) + 1\right]}{2} - \frac{1}{q_1^2} \left(\frac{A_{v_4} D_4}{K} + \frac{A_4 D_4}{K} + C_{t_4} D_4 \right) \right] -\lambda_4 + \lambda_5 = 0$$

$$\frac{2}{6} \left\{ \frac{h_2}{2} + \frac{h_{v_2} \left[K(1 - \left(\frac{D_2}{P_3}\right) + 1 \right]}{2} - \frac{1}{q_2^2} \left(\frac{A_{v_3} D_3}{K} + \frac{A_3 D_3}{K} + C_{t_3} D_3 \right) \right\}$$

$$-\lambda_{1} + \lambda_{2} = 0$$

$$\frac{2}{6} \left\{ \frac{h_{3}}{2} + \frac{h_{v_{3}} \left[K(1 - \left(\frac{D_{3}}{P_{2}}\right) + 1\right]}{2} - \frac{1}{q_{3}^{2}} \left(\frac{A_{v_{2}}D_{2}}{K} + \frac{A_{2}D_{2}}{K} + C_{t_{2}}D_{2}\right) - \lambda_{1} + \lambda_{2} = 0 \right\}$$

$$\frac{1}{6} \left\{ \frac{h_4}{2} + \frac{h_{v_4} \left[K(1 - \left(\frac{D_4}{P_1}\right) + 1\right]}{2} - \frac{1}{q_4^2} \left(\frac{A_{v_1} D_1}{K} + \frac{A_1 D_1}{K} + C_{v_1} D_1\right) \right\} - \lambda_3 = 0$$

$$\lambda_{1}(q_{2} - q_{1}) = 0$$

$$\lambda_{2}(q_{3} - q_{2}) = 0$$

$$\lambda_{3}(q_{4} - q_{3}) = 0$$

$$\lambda_{4}q_{1} = 0$$

Because $q_1 > 0$ and $\lambda_4 q_1 = 0$ then $\lambda_4 = 0$. If $\lambda_1 = \lambda_2 = \lambda_3 = 0$ then $q_4 < q_3 < q_2 < q_1$, it does not satisfy the constraints $0 < q_1$ $\leq q_2 \leq q_3 \leq q_4$. Therefore $q_2 = q_1$, $q_3 = q_2 \& q_4 = q_3$.

(ie) $q_1 = q_2 = q_3 = q_4 = q^*$.

Hence from we find the optimal order quantity q_{NL}^* as

$$q_{NL}^{*} = \begin{cases} \left[\left\{ A_{v_{1}}D_{1} + 2A_{v_{2}}D_{2} + 2A_{v_{3}}D_{3} + A_{v_{4}}D_{4} \right\} \\ + \left\{ A_{1}D_{1} + 2A_{2}D_{2} + 2A_{3}D_{3} + A_{4}D_{4} \right\} \\ + \left\{ C_{t_{1}}KD_{1} + 2C_{t_{2}}KD_{2} + 2C_{t_{3}}KD_{3} + C_{t_{4}}KD_{4} \right\} \right] 2 \\ K \left[\left\{ h_{v_{1}} \left[K \left(1 - \frac{D_{1}}{P_{4}} \right) + 1 \right] \right. \\ + 2h_{v_{2}} \left[K \left(1 - \frac{D_{2}}{P_{3}} \right) + 1 \right] + 2h_{v_{3}} \left[K \left(1 - \frac{D_{3}}{P_{2}} \right) + 1 \right] \\ + h_{v_{4}} \left[K \left(1 - \frac{D_{4}}{P_{1}} \right) + 1 \right] \right\} + \left\{ h_{1} + 2h_{2} + 2h_{3} + h_{4} \right\} \right] \end{cases}$$

IV. NUMERICAL EXAMPLES

In this section, a numerical example is shown to illustrate the model. Let the

Production rate, P = 19,200 units/year, Demand Rate, D = 4,800 units/year, Vendor setup cost, $A_v = \$600$ /cycle, Ordering cost of buyer, A = \$25/order, Vendor holding cost, $h_v = \$6$ /unit/year, Buyer holding cost, h = \$7/unit/year, Buyer's transportation cost, $C_t = \$50$ /delivery, Number of delivery, K = 6 The optimal solution is q = 192.354 TUC_{NL} = 7694.15

Suppose Fuzzy production rate P is "more or less than 19200"

 $\tilde{\mathbf{P}} = (18800, 19000, 19400, 19600)$

Fuzzy annual demand rate D is "more or less than 4800"

 $\tilde{D} = (4200, 4500, 5100, 5400)$

Fuzzy setup cost A_{ν} for vendor per cycle is "more or less than 600"

 $A_v = (400, 500, 700, 800)$

Fuzzy ordering cost of buyer per order A is "more or less than 25"

 $\tilde{A} = (15, 20, 30, 35)$

Fuzzy holding cost h_{ν} for vendor per unit per year is "more or less than 6"

 $\tilde{h}_{v} = (4, 5, 7, 8)$

Fuzzy holding cost h for buyer per unit per year is "more or less than 7"

 $\hat{h} = (3, 5, 9, 11)$

Fuzzy transportation cost C_t for buyer per delivery is "more or less than 50"

 $\tilde{C}_{t} = (30, 40, 60, 70)$

If the number of delivery k is 6, then fuzzy order quantity

 $q_{NL}^* = (179.49, 185.75, 199.14, 206.04)$

The optimal total cost when the production down time is bigger than the upper bound of the machine unavailability time is

 $\tilde{\text{TUC}}_{\text{NL}}^* = (4641.012821, 6136.485317, 9305.045038, 10942.08398)$

V. CONCLUSION

In this paper, the machine unavailability time is assumed to be uniformly distributed. The numerical example illustrates how the multiple deliveries result in a lower cost than the single delivery model. The stochastic machine time model results in a higher cost and more delivery frequencies when compared to a perfect machine model. The proposed model helps enterprises to optimize their profit by coordinating the number of deliveries for various machine unavailability time and lost sales cost. For each fuzzy model, a method of defuzzification namely the Graded Mean Integration Representation is employed to find the estimate of total cost function in the fuzzy sense and then the corresponding optimal order lotsize is derived from Kuhn-Tucker Model.

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