# Fuzzy Inventory Model : Imperfective Items Under One-Time-Only Discount

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*Abstract :* **In practice, when a supplier wants to succeed in the competitive business world, unexpected inventory in the factory or change in the production run of the product, he/she may offer a special price discount to motivate buyers to order a special quantity. In this paper, we have introduced the fuzzy economic order quantity model with imperfective items under a one-time-only discount. Two fuzzy inventory models are discussed with fuzzy parameters for crisp order quantity or fuzzy order quantity in order to extend the order quantity inventory model to the fuzzy environment. We use function principle as arithmetical operations of fuzzy annual inventory cost, and use the graded mean integration representation method to defuzzify the fuzzy total annual inventory cost. Then we use the Lagrangean method to find the optimal economic order quantity of the fuzzy inventory model. Furthermore, a numerical example is provided and the results of fuzzy and crisp models are compared.**

*Keywords :* **Function principle, one-time-only discount, imperfect quality, graded mean integration representation.**

## I. INTRODUCTION

To make the imperfective product to reach the public easily, the manufacture do the advertises with one-time-only discount. This leads to attract the people to make use of the product consistently.

Since the olden days onwards the EOQ and EPQ models are popular and being used for the inventory management. The EPQ is the next version of the EOQ model, replacing the assumptions of simultaneous replenishment. However, one of the weaknesses of current inventory models is the unrealistic assumption that all items produced are of good quality [1]. Defective items, as a result of considering imperfect quality production process were initially considered by Rosenblatt and Lee [2], Porteus [3].

Recently, Salameh and Jaber [4] who presented a model with imperfect rate to be a random variable and all the imperfective items can be shipped in a single lot at a discounted price. Cardenas – Barron [5] corrected a mistake that appeared in the work of Salameh and Jaber. Goyal and Cardenas – Barron [6] reconsidered the work done in [4] and presented a practical approach for determining the optimal lot size. H.C. Chang [7] developed EOQ model having imperfect quality without shortages in which demand and defective rate and then derived the corresponding optimal lot size. H.M. Wee et al [8] extended the inventory model having imperfect quality with shortage backordering. Aucamp and Kuzdrall [8] considered an economic order quantity for a one-time-only discount and the impending price increase by minimizing discounted cash flows. They also gave two situations which are frequently faced in explain the reasonableness of the issue of only tine only discounts. W-K Kevin Hsu, H-F. Yu[10] investigate this paper is to investigate an EOQ model with imperfective items under a one-time-only sale, where the defectives can be screened out by a 100% screening process and then can be sold in a single batch by the end of the 100% screening process.

In this paper, an inventory model having imperfect items under one-time-only discount where demand, defective rate, ordering cost, purchasing cost, and screening rate is trapezoidal fuzzy number. Two inventory models are discussed. The first model is fuzzy inventory model for crisp order quantity. The second model is fuzzy order quantity. To find the economic order quantity in the fuzzy sense and then derive the corresponding optimal lot size.

### 1.1. Notation

The notation used in this paper is in the following.

- $\lambda$ the demand rate
- c the purchasing cost per unit
- b the holding cost rate per unit / per unit time
- a the ordering cost per order
- p the defective percentage
- s the screening rate, $s > \lambda$
- w the screening cost per unit
- y ordering quantity
- * the super script representing optimal value

## II. CRISP MODEL

Pictorially, Salameh and Jaber's Model [4] can be illustrated as Fig.1, where y $(1-p)/\lambda$ is the cycle length, $y_p$ is the number of imperfective items with drawn form inventory and y/s is the total screening time of y unit ordered per cycle.

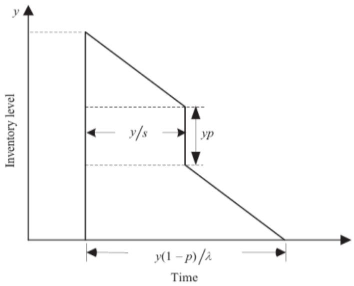


Fig.1. The behavior of the inventory level over time for Salameh & Jaber's model

For a given percentage of imperfective items in each y. The total cost per cycle, C(y), consists of procurement cost, screening cost and holding cost.

$$C(y) = \left(a + cy\right) + wy + cb\left[\frac{y^2(1-p)^2}{2\lambda} + \frac{y^2 p}{s}\right]$$

Since the cycle length if y (1-p)/λ, the annual cost T(y) can be derived as

$$T(y) = \left\{\left(a + cy\right) + wy + cb\left[\frac{y^2(1-p)^2}{2\lambda} + \frac{y^2 p}{s}\right]\right\} \times \frac{\lambda}{y(1-p)}$$

Also, the optimal y then can be found as

$$y^* = \sqrt{\frac{2a\,.\,\lambda\,.\,s}{\left[b\left[s(1-p)^2 + 2\lambda p\right]\right]}}$$

In case, all items are of perfect quantity the result becomes i.e. p = 0 then the $y^*$ will reduce to $\sqrt{\frac{2a\lambda}{c\,b}}$ the traditional EOQ.

## III. METHODOLOGY

### *3.1. Graded Mean Integration Representation Method*

Chen and Hsieh introduced Graded Mean Integration Representation method based on the integral value of graded mean h-level of generalized fuzzy number for defuzzifying generalized fuzzy number.

Suppose A is a generalized fuzzy number as shown in Fig.2. It is described as any fuzzy subset of the real line R, whose membership function, $\mu_{\tilde{A}}$ satisfies the following conditions:

1. $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0, 1].
2. $\mu_{\tilde{A}}(x) = 0$, $-\infty < x \le a_1$,
3. $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$,
4. $\mu_{\tilde{A}}(x) = w_A$, $a_2 \le x \le a_3$,
5. $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$,
6. $\mu_{\tilde{A}}(x) = 0$, $a_4 \le x < \infty$,

where $0 < w_A \le 1$, and $a_1$, $a_2$, $a_3$ and $a_4$ are real numbers.

Also this type of generalized fuzzy number be denoted as $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)LR$. When $w_A = 1$, it can be simplified as $\tilde{A} = (a_1, a_2, a_3, a_4)$ LR.

Second, by Graded Mean Integration Representation Method $L^{-1}$ and $R^{-1}$ are the inverse functions of L and R, respectively, and the graded mean h-level

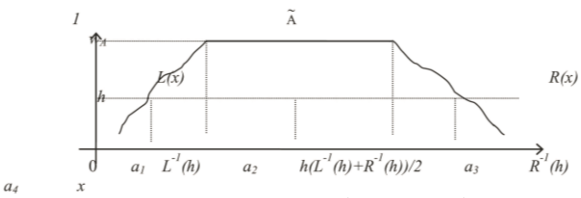


Fig.2. The graded mean h-level value of generalized fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$



value of generalized fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)L$ is $h(L^{-1}(h) + R^{-1}(h))/2$. Then Graded Mean Integration Representation of A and P ( $\tilde{A}$ ) with grate $w_A$, where

$$P(\tilde{A}) = \int_0^{w_A} h\left(\frac{L^{-1}(h)+R^{-1}(h)}{2}\right)dh \Big/ \int_0^{w_A} h\,dh$$

with $0 < h \le w_A$ and $0 < w_A \le 1$.

Throughout this paper, we have only use popular trapezoidal fuzzy number as the type of all fuzzy parameters in our proposed fuzzy production inventory models. Let $\tilde{B}$ be a trapezoidal fuzzy number, and be denoted as $\tilde{B} = (b_1, b_2, b_3, b_4)$. Then we can get the Graded Mean Integration Representation of $\tilde{B}$ by formula (1) as

$$P(\tilde{B}) = \int_0^1 h\left(\frac{b_1+b_4+(b_2-b_1-b_4+b_3)h}{2}\right)dh \Big/ \int_0^1 h\,dh$$
$$= \frac{b_1+2b_2+2b_3+b_4}{6}$$

### *3.2. The Fuzzy Arithmetical Operations under Function Principle*

*The fuzzy arithmetical operations under function principle*

In [11], Function principle is proposed to be as the fuzzy arithmetical operations by trapezoidal fuzzy numbers. We describe some fuzzy arithmetical operations under Function principle as follows:

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$

and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two fuzzy trapezoidal fuzzy numbers. Then

1. The addition of $\tilde{A}$ and $\tilde{B}$ is
   $\tilde{A} \oplus \tilde{B} = (c_1, c_2, c_3, c_4)$
   where $T = \{a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4\}$
   where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and $b_4$ are any real numbers.
2. The multiplication of $\tilde{A}$ and $\tilde{B}$ is
   $\tilde{A} \oplus \tilde{B} = (c_1, c_2, c_3, c_4)$
   where $T = \{a_1b_1, a_2b_2, a_3b_3, a_4b_4\}$
   $T_1 = \{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$, $C_1 = \min T$, $C_2 = \min T_1$, $C_3 = \max T$, $C_4 = \max T_1$.
   If $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and $b_4$ are all non zero positive real numbers, then $\tilde{A} \otimes \tilde{B} = \{a_1b_1, a_2b_2, a_3b_3, a_4b_4\}$
3. $-\tilde{B} = (-b_4, -b_3, -b_2, -b_1)$, then the subtraction of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \ominus \tilde{B} = \{a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1\}$
   where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and $b_4$ are any real numbers.
4. $\frac{1}{\tilde{B}} = \tilde{B}^{-1} = \left(\frac{1}{b_4}, \frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}\right)$ Where $b_1, b_2, b_3$ and $b_4$ are any real numbers. If $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and $b_4$ are all non zero positive real numbers, then the division of $\tilde{A}$ and $\tilde{B}$ is
   $\tilde{A} \oslash \tilde{B} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right)$
5. Let $x \in R$. Then
   (i) $a \ge 0$, $\alpha \otimes \tilde{A} = (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4)$

(ii) $a < 0$, $\alpha \otimes \tilde{A} = (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1)$

## IV. FUZZY MODEL

### *4.1. The Fuzzy Inventory Model for Crisp Order Quantity*

Throughout this paper, we use of the following variables in order to simplify the treatment of the fuzzy order inventory model.

| | |
|---|---|
| $\tilde{\lambda}$ | Fuzzy demand rate |
| $\tilde{C}$ | Fuzzy purchasing cost per unit |
| $\tilde{B}$ | Fuzzy holding cost rate per unit / per unit time |
| $\tilde{A}$ | Fuzzy ordering cost per order |
| $\tilde{P}$ | Fuzzy defective percentage |
| $\tilde{S}$ | Fuzzy screening rate, $\tilde{S} > \tilde{\lambda}$ |
| $\tilde{W}$ | Fuzzy screening cost per unit |
| $y_p$ | Crisp order quantity |
| $\tilde{y}_p$ | Fuzzy order quantity |

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$, $\tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$, $\tilde{P} = (p_1, p_2, p_3, p_4)$, $\tilde{C} = (c_1, c_2, c_3, c_4)$, $\tilde{W} = (w_1, w_2, w_3, w_4)$, $\tilde{B} = (b_1, b_2, b_3, b_4)$ and $\tilde{S} = (s_1, s_2, s_3, s_4)$.

The annual cost T(y) is fuzzy sense is given by

$$T(\tilde{y}) = \Big[\big(\tilde{A} \otimes \tilde{\lambda}\big) \oslash \big(y_p \otimes (1 - \tilde{P})\big) \oplus \big((\tilde{C} \otimes \tilde{\lambda}) \oslash (1 - \tilde{P})\big) \oplus \big((\tilde{W} \otimes \tilde{\lambda}) \oslash (1 - \tilde{P})\big) \oplus \big((\tilde{C} \otimes \tilde{B} \otimes y_p \otimes (1 - \tilde{P})) \oslash 2\big) \oplus \big((\tilde{C} \otimes \tilde{B} \otimes y_p \otimes \tilde{P} \otimes \tilde{\lambda}) \oslash (\tilde{S} \otimes (1 - \tilde{P}))\big)\Big]$$

Where $\oslash$, $\otimes$, $\oplus$ and $\ominus$ are the fuzzy arithmetical operations under function principle

$$T(\tilde{y}) = \Bigg[\frac{a_1\lambda_1}{y_p(1-p_1)} + \frac{c_1\lambda_1}{(1-p_1)} + \frac{w_1\lambda_1}{(1-p_1)} + \frac{c_1b_1y_p(1-p_1)}{2} + \frac{c_1b_1y_pp_1\lambda_1}{s_4(1-p_1)},$$
$$\frac{a_2\lambda_2}{y_p(1-p_2)} + \frac{c_2\lambda_2}{(1-p_2)} + \frac{w_2\lambda_2}{(1-p_2)} + \frac{c_2b_2y_p(1-p_2)}{2} + \frac{c_2b_2y_pp_2\lambda_2}{s_3(1-p_2)},$$
$$\frac{a_3\lambda_3}{y_p(1-p_3)} + \frac{c_3\lambda_3}{(1-p_3)} + \frac{w_3\lambda_3}{(1-p_3)} + \frac{c_3b_3y_p(1-p_3)}{2} + \frac{c_3b_3y_pp_3\lambda_3}{s_2(1-p_3)},$$
$$\frac{a_4\lambda_4}{y_p(1-p_4)} + \frac{c_4\lambda_4}{(1-p_4)} + \frac{w_4\lambda_4}{(1-p_4)} + \frac{c_4b_4y_p(1-p_4)}{2} + \frac{c_4b_4y_pp_4\lambda_4}{s_1(1-p_4)}\Bigg]$$

$P(T(\tilde{y})) =$

$$\frac{1}{6}\Bigg[\Bigg[\frac{a_1\lambda_1}{y_p(1-p_1)} + \frac{c_1\lambda_1}{(1-p_1)} + \frac{w_1\lambda_1}{(1-p_1)} + \frac{c_1b_1y_p(1-p_1)}{2} + \frac{c_1b_1y_pp_1\lambda_1}{s_4(1-p_1)}\Bigg] +$$
$$\frac{a_2\lambda_2}{y_p(1-p_2)} + \frac{c_2\lambda_2}{(1-p_2)} + \frac{w_2\lambda_2}{(1-p_2)} + \frac{c_2b_2y_p(1-p_2)}{2} + \frac{c_2b_2y_pp_2\lambda_2}{s_3(1-p_2)},$$
$$\frac{a_3\lambda_3}{y_p(1-p_3)} + \frac{c_3\lambda_3}{(1-p_3)} + \frac{w_3\lambda_3}{(1-p_3)} + \frac{c_3b_3y_p(1-p_3)}{2} + \frac{c_3b_3y_pp_3\lambda_3}{s_2(1-p_3)},$$
$$\frac{a_4\lambda_4}{y_p(1-p_4)} + \frac{c_4\lambda_4}{(1-p_4)} + \frac{w_4\lambda_4}{(1-p_4)} + \frac{c_4b_4y_p(1-p_4)}{2} + \frac{c_4b_4y_pp_4\lambda_4}{s_1(1-p_4)}\Bigg]$$

We can get the optimal order quantity $y_p^*$ when $P(T(\tilde{y}))$ is minimized. By taking the first derivative of $P(T(\tilde{y}))$ with respect to $y_p$.

$$\frac{\partial P(T(\tilde{y}))}{\partial y_p} = \frac{1}{6}\Bigg[-\frac{1}{y_p^2}\left(\frac{a_1\lambda_1}{(1-p_1)} + 2\frac{a_2\lambda_2}{(1-p_2)} + 2\frac{a_3\lambda_3}{(1-p_3)} + \frac{a_4\lambda_4}{(1-p_4)}\right)$$
$$+\frac{1}{2}[c_1b_1(1-p_4) + 2c_2b_2(1-p_3) + 2c_3b_3(1-p_2) + c_4b_4(1-p_1)]$$
$$+\left(\frac{c_1b_1p_1\lambda_1}{s_4(1-p_1)} + 2\frac{c_2b_2p_2\lambda_2}{s_3(1-p_2)} + 2\frac{c_3b_3p_3\lambda_3}{s_2(1-p_3)} + \frac{c_4b_4p_4\lambda_4}{s_1(1-p_4)}\right)\Bigg]$$

With respect to $\dfrac{\partial P(T(\tilde{y}))}{\partial y_p} = 0$

$$y_p^* = \sqrt{\frac{2\left[\left(\frac{a_1\lambda_1}{(1-p_1)} + 2\frac{a_2\lambda_2}{(1-p_2)} + 2\frac{a_3\lambda_3}{(1-p_3)} + \frac{a_4\lambda_4}{(1-p_4)}\right)\right]}{c_1b_1(1-p_4)+2c_2b_2(1-p_3)+2c_3b_3(1-p_2)+c_4b_4(1-p_1) + 2\left[\left(\frac{c_1b_1p_1\lambda_1}{s_4(1-p_1)} + 2\frac{c_2b_2p_2\lambda_2}{s_3(1-p_2)} + 2\frac{c_3b_3p_3\lambda_3}{s_2(1-p_3)} + \frac{c_4b_4p_4\lambda_4}{s_1(1-p_4)}\right)\right]}}$$

### *4.2. The fuzzy inventory model for fuzzy order quantity*

In this section, we discuss the fuzzy order inventory model. Here by changing the crisp order quantity into fuzzy order quantity. The fuzzy order quantity $\tilde{y}_p$ be a trapezoidal fuzzy number.

i.e. $\tilde{y}_p = \left(y_{p_1}, y_{p_2}, y_{p_3}, y_{p_4}\right)$ with

$0 < y_{p_1} \le y_{p_2} \le y_{p_3} \le y_{p_4}$

The fuzzy annual cost

$$T(\tilde{y}) = \Big[\big(\tilde{A} \otimes \tilde{\lambda}\big) \oslash \big(\tilde{y}_p \otimes (1 - \tilde{P})\big) \oplus \big((\tilde{C} \otimes \tilde{\lambda}) \oslash (1 - \tilde{P})\big) \oplus \big((\tilde{W} \otimes \tilde{\lambda}) \oslash (1 - \tilde{P})\big) \oplus \big((\tilde{C} \otimes \tilde{B} \otimes \tilde{y}_p \otimes (1 - \tilde{P})) \oslash 2\big) \oplus \big((\tilde{C} \otimes \tilde{B} \otimes \tilde{y}_p \otimes \tilde{P} \otimes \tilde{\lambda}) \oslash (\tilde{S} \otimes (1 - \tilde{P}))\big)\Big]$$

$T(\tilde{y}) =$

$$\left[\frac{a_1\lambda_1}{y_{p_4}(1-p_1)} + \frac{c_1\lambda_1}{(1-p_1)} + \frac{w_1\lambda_1}{(1-p_1)} + \frac{c_1b_1y_{p_1}(1-p_1)}{2} + \frac{c_1b_1y_{p_1}p_1\lambda_1}{s_4(1-p_1)},\right.$$
$$\frac{a_2\lambda_2}{y_{p_3}(1-p_2)} + \frac{c_2\lambda_2}{(1-p_2)} + \frac{w_2\lambda_2}{(1-p_2)} + \frac{c_2b_2y_{p_2}(1-p_2)}{2} + \frac{c_2b_2y_{p_2}p_2\lambda_2}{s_3(1-p_2)},$$
$$\frac{a_3\lambda_3}{y_{p_2}(1-p_3)} + \frac{c_3\lambda_3}{(1-p_3)} + \frac{w_3\lambda_3}{(1-p_3)} + \frac{c_3b_3y_{p_3}(1-p_3)}{2} + \frac{c_3b_3y_{p_3}p_3\lambda_3}{s_2(1-p_3)},$$
$$\left.\frac{a_4\lambda_4}{y_{p_1}(1-p_4)} + \frac{c_4\lambda_4}{(1-p_4)} + \frac{w_4\lambda_4}{(1-p_4)} + \frac{c_4b_4y_{p_4}(1-p_4)}{2} + \frac{c_4b_4y_{p_4}p_4\lambda_4}{s_1(1-p_4)}\right]$$

Also, we can obtain the graded mean integration representation of $T(\tilde{y})$.

$P(T(\tilde{y})) =$

$$\frac{1}{6}\left[\left[\frac{a_1\lambda_1}{y_{p_4}(1-p_1)} + \frac{c_1\lambda_1}{(1-p_1)} + \frac{w_1\lambda_1}{(1-p_1)} + \frac{c_1b_1y_{p_1}(1-p_4)}{2} + \frac{c_1b_1y_{p_1}p_1\lambda_1}{s_4(1-p_1)}\right]\right.$$
$$+2\left[\frac{a_2\lambda_2}{y_{p_3}(1-p_2)} + \frac{c_2\lambda_2}{(1-p_2)} + \frac{w_2\lambda_2}{(1-p_2)} + \frac{c_2b_2y_{p_2}(1-p_2)}{2} + \frac{c_2b_2y_{p_2}p_2\lambda_2}{s_3(1-p_2)}\right]$$
$$+2\left[\frac{a_3\lambda_3}{y_{p_2}(1-p_3)} + \frac{c_3\lambda_3}{(1-p_3)} + \frac{w_3\lambda_3}{(1-p_3)} + \frac{c_3b_3y_{p_3}(1-p_3)}{2} + \frac{c_3b_3y_{p_3}p_3\lambda_3}{s_2(1-p_3)}\right]$$
$$\left.+\left[\frac{a_4\lambda_4}{y_{p_1}(1-p_4)} + \frac{c_4\lambda_4}{(1-p_4)} + \frac{w_4\lambda_4}{(1-p_4)} + \frac{c_4b_4y_{p_4}(1-p_1)}{2} + \frac{c_4b_4y_{p_4}p_4\lambda_4}{s_1(1-p_4)}\right]\right]$$

with $0 < y_{p_1} \le y_{p_2} \le y_{p_3} \le y_{p_4}$.

If we replace inequality conditions $0 < y_{p_1} \le y_{p_2} \le y_{p_3} \le y_{p_4}$ into the following inequality.

$y_{p_2} - y_{p_1} \ge 0$, $y_{p_3} - y_{p_2} \ge 0$, $y_{p_4} - y_{p_3} \ge 0$ and $y_{p_1} > 0$.

Extension of the Lagrangean Method is used to find the solutions of $y_{p_1}, y_{p_2}, y_{p_3}, y_{p_4}$ to minimize $P(T(\tilde{y}))$.

*Step 1:* Solve the unconstraint problem
Minimize

$P(T(\tilde{y})) =$

$$\frac{1}{6}\left[\left[\frac{a_1\lambda_1}{y_{p_4}(1-p_1)} + \frac{c_1\lambda_1}{(1-p_1)} + \frac{w_1\lambda_1}{(1-p_1)} + \frac{c_1b_1y_{p_1}(1-p_4)}{2} + \frac{c_1b_1y_{p_1}p_1\lambda_1}{s_4(1-p_1)}\right]+\right.$$
$$2\left[\frac{a_2\lambda_2}{y_{p_3}(1-p_2)} + \frac{c_2\lambda_2}{(1-p_2)} + \frac{w_2\lambda_2}{(1-p_2)} + \frac{c_2b_2y_{p_2}(1-p_2)}{2} + \frac{c_2b_2y_{p_2}p_2\lambda_2}{s_3(1-p_2)}\right]+$$
$$2\left[\frac{a_3\lambda_3}{y_{p_2}(1-p_3)} + \frac{c_3\lambda_3}{(1-p_3)} + \frac{w_3\lambda_3}{(1-p_3)} + \frac{c_3b_3y_{p_3}(1-p_3)}{2} + \frac{c_3b_3y_{p_3}p_3\lambda_3}{s_2(1-p_3)}\right]+$$
$$\left.\left[\frac{a_4\lambda_4}{y_{p_1}(1-p_4)} + \frac{c_4\lambda_4}{(1-p_4)} + \frac{w_4\lambda_4}{(1-p_4)} + \frac{c_4b_4y_{p_4}(1-p_1)}{2} + \frac{c_4b_4y_{p_4}p_4\lambda_4}{s_1(1-p_4)}\right]\right]$$

To find the minimization of $P(T(\tilde{y}))$, by taking the first derivative of $P(T(\tilde{y}))$ with separate to $y_{p_1}, y_{p_2}, y_{p_3}, y_{p_4}$ then let all the partial derivatives equal to zero.

We get

$$y_{p_1} = \sqrt{\frac{2\left(\frac{a_4\lambda_4}{1-p_4}\right)}{c_1b_1(1-p_4) + 2\frac{c_1b_1p_1\lambda_1}{s_4(1-p_1)}}}$$

$$y_{p_2} = \sqrt{\frac{4\left(\frac{a_3\lambda_3}{1-p_3}\right)}{2c_2b_2(1-p_3) + 4\frac{c_2b_2p_2\lambda_2}{s_3(1-p_2)}}}$$

$$y_{p_3} = \sqrt{\frac{4\left(\frac{a_2\lambda_2}{1-p_2}\right)}{2c_3b_3(1-p_2) + 4\frac{c_3b_3p_3\lambda_3}{s_2(1-p_3)}}}$$

$$y_{p_4} = \sqrt{\frac{2\left(\frac{a_1\lambda_1}{1-p_1}\right)}{c_4b_4(1-p_1) + 2\frac{c_4b_4p_4\lambda_4}{s_1(1-p_4)}}}$$

But $y_{p_1} > y_{p_2} > y_{p_3} > y_{p_4}$. It does not satisfy the constraint $0 < y_{p_1} \le y_{p_2} \le y_{p_3} \le y_{p_4}$ therefore K = 1 and go to step 2.

*Step 2 :* Convert inequality constraint into equality constraint and optimize $P(T(\tilde{y}))$ subject to equality constraint is $y_{p_2} - y_{p_1} = 0$ by the Lagrangean Method. We have $L(y_{p_1}, y_{p_2}, y_{p_3}, y_{p_4}, \lambda) = P(T(\tilde{y})) - \lambda(y_{p_2} - y_{p_1})$.

To find the minimization $L(y_{p_1}, y_{p_2}, y_{p_3}, y_{p_4}, \lambda)$ of the derivative of $L(y_{p_1}, y_{p_2}, y_{p_3}, y_{p_4}, \lambda)$ with respect to $y_{p_1}, y_{p_2}, y_{p_3}, y_{p_4}$ and $\lambda$.

Let all the partial derivatives equal to zero and solve $y_{p_1}, y_{p_2}, y_{p_3}, y_{p_4}$ then we get

$y_{p_1} = y_{p_2} =$

$$\sqrt{\frac{2\left[\frac{a_4\lambda_4}{(1-p_4)} + 2\frac{a_3\lambda_3}{(1-p_3)}\right]}{c_1b_1(1-p_4) + 2c_2b_2(1-p_3) + 2\left[\left(\frac{c_1b_1p_1\lambda_1}{s_4(1-p_1)} + 2\frac{c_2b_2p_2\lambda_2}{s_3(1-p_2)}\right)\right]}}$$

$$y_{p_3} = \sqrt{\frac{4\frac{a_2\lambda_2}{1-p_2}}{2c_3b_3(1-p_2) + 4\frac{c_3b_3p_3\lambda_3}{s_2(1-p_3)}}}$$

$$y_{p_4} = \sqrt{\dfrac{2\dfrac{a_1\lambda_1}{1-p_1}}{c_4b_4(1-p_1)+2\dfrac{c_4b_4p_4\lambda_4}{s_1(1-p_4)}}}$$

Here $y_{p_1} > y_{p_4}$, it does not satisfy the constraint $0 < y_{p_1} \le y_{p_2} \le y_{p_3} \le y_{p_4}$. Therefore it is not a local optimum. Therefore K = 2 and go to step 3.

*Step 3 :* If we select any other one inequality constraint to be equality constraint. Convert the inequality constraints $y_{p_2} - y_{p_1} \ge 0$ and $y_{p_3} - y_{p_2} \ge 0$ into inequality constraints $y_{p_2} - y_{p_1} = 0$ and $y_{p_3} - y_{p_1} = 0$. we optimize $P(T(\tilde{y}))$ subject to the constraints $y_{p_2} - y_{p_1} = 0$ ; $y_{p_3} - y_{p_2} = 0$ by Lagrangean Method.

We construct the Lagrangean function

$$L(y_{p_1}, y_{p_2}, y_{p_3}, y_{p_4}, \lambda_1, \lambda_2) = P(T(\tilde{y})) - \lambda_1(y_{p_2} - y_{p_1}) - \lambda_2(y_{p_3} - y_{p_2}).$$

By taking the partial derivatives of $L(y_{p_1}, y_{p_2}, y_{p_3}, y_{p_4}, \lambda_1, \lambda_2, \lambda_3)$ with respect to $y_{p_1}, y_{p_2}, y_{p_3}, y_{p_4}, \lambda_1, \lambda_2$ and $\lambda_3$.

Let all the partial derivatives equal to zero and solve $y_{p_1}, y_{p_2}, y_{p_3}$ and $y_{p_4}$ we get

$$\therefore\ y_1 = y_2 = y_3 = \sqrt{\dfrac{2\left[\dfrac{a_4\lambda_4}{(1-p_4)} + 2\dfrac{a_3\lambda_3}{(1-p_3)} + + 2\dfrac{a_2\lambda_2}{(1-p_2)}\right]}{c_1b_1(1-p_4) + 2c_2b_2(1-p_3) + 2c_3b_3(1-p_2) + 2\left[\left(\dfrac{c_1b_1p_1\lambda_1}{s_4(1-p_1)} + 2\dfrac{c_2b_2p_2\lambda_2}{s_3(1-p_2)} + 2\dfrac{c_3b_3p_3\lambda_3}{s_2(1-p_3)}\right)\right]}}$$

$$\frac{\partial L}{\partial y_{p_4}} = 0$$

$$y_{p_4} = \sqrt{\dfrac{\dfrac{2a_1\lambda_1}{(1-p_1)}}{c_4b_4(1-p_1) + \dfrac{2c_4b_4p_4\lambda_4}{s_1(1-p_4)}}}$$

But $y_{p_1} > y_{p_4}$ it doesn't satisfy the constraint $0 < y_{p_1} \le y_{p_2} \le y_{p_3} \le y_{p_4}$, therefore it is not a local optimum. Set K = 3 and go to step 4.

*Step 4 :* Convert the inequality constraints into the equality constraints Inequality constraints is

$$y_{p_2} - y_{p_1} \ge 0,\ y_{p_3} - y_{p_2} \ge 0 \text{ and } y_{p_4} - y_{p_3} \ge 0.$$

Equality constraints is

$$y_{p_2} - y_{p_1} = 0,\ y_{p_3} - y_{p_2} = 0 \text{ and } y_{p_4} - y_{p_3} = 0.$$

We construct the Lagrangean function

$$L(y_{p_1}, y_{p_2}, y_{p_3}, y_{p_4}, \lambda_1, \lambda_2, \lambda_3) = P(T(\tilde{y})) - \lambda_1(y_{p_2} - y_{p_1}) - \lambda_2(y_{p_3} - y_{p_2}) - \lambda_3(y_{p_4} - y_{p_3}).$$

By taking the partial derivatives of $L(y_{p_1}, y_{p_2}, y_{p_3}, y_{p_4}, \lambda_1, \lambda_2, \lambda_3)$ with respect to $y_{p_1}, y_{p_2}, y_{p_3}, y_{p_4}, \lambda_1, \lambda_2$ and $\lambda_3$.

Let all the partial derivatives equal to zero and solve $y_{p_1}, y_{p_2}, y_{p_3}$ and $y_{p_4}$. Then we get,

$$\therefore\ y_1 = y_2 = y_3 = y_4 = \sqrt{\dfrac{2\left[\dfrac{a_4\lambda_4}{(1-p_4)} + 2\dfrac{a_3\lambda_3}{(1-p_3)} + 2\dfrac{a_2\lambda_2}{(1-p_2)} + \dfrac{a_1\lambda_1}{(1-p_1)}\right]}{c_1b_1(1-p_4)+2c_2b_2(1-p_3)+2c_3b_3(1-p_2)+c_4b_4(1-p_1) + 2\left[\left(\dfrac{c_1b_1p_1\lambda_1}{s_4(1-p_1)} + 2\dfrac{c_2b_2p_2\lambda_2}{s_3(1-p_2)} + 2\dfrac{c_3b_3p_3\lambda_3}{s_2(1-p_3)} + \dfrac{c_4b_4p_4\lambda_4}{s_1(1-p_4)}\right)\right]}}$$

The above result satisfies all inequality constraints. Therefore it is a local optimum solution

Let $y_{p_1} = y_{p_2} = y_{p_3} = y_{p_4} = y_p$.

Then the optimal fuzzy order quantity is $Y_p^* = (y_p^*, y_p^*, y_p^*, y_p^*)$, where

$$y_p^* = \sqrt{\dfrac{2\left[\dfrac{a_4\lambda_4}{(1-p_4)} + 2\dfrac{a_3\lambda_3}{(1-p_3)} + 2\dfrac{a_2\lambda_2}{(1-p_2)} + \dfrac{a_1\lambda_1}{(1-p_1)}\right]}{c_1b_1(1-p_4) + 2c_2b_2(1-p_3) + 2c_3b_3(1-p_2) + c_4b_4(1-p_1) + 2\left[\left(\dfrac{c_1b_1p_1\lambda_1}{s_4(1-p_1)} + 2\dfrac{c_2b_2p_2\lambda_2}{s_3(1-p_2)} + 2\dfrac{c_3b_3p_3\lambda_3}{s_2(1-p_3)} + \dfrac{c_4b_4p_4\lambda_4}{s_1(1-p_4)}\right)\right]}}$$

## V. NUMERICAL EXAMPLE

### <u>5.1 Crisp Model</u>

| | | |
|---|---|---|
| Demand rate, $\lambda$ | = | 8000units/year |
| Purchasing cost per unit, c | = | 12/unit |
| Holding cost rate per unit, b | = | 0.1/unit |
| Ordering cost, a | = | 80/cycle |
| Screening rate, s | = | 24000units/year |
| Defective percentage, p | = | 0.1 |
| Optimal order quantity T(y) | = | 1103.05 |

### <u>5.2 Fuzzy Model</u>

Let $\tilde{\lambda}$ = (7900, 8000, 8000, 8100)

$\tilde{C}$ = (11, 11.5, 12.5, 13)

$\tilde{B}$ = (0.095, 0.1, 0.1, 0.105 )

$\tilde{A}$ = (79.5, 80, 80, 80.5 )

$\tilde{S}$ = (23000, 24000, 24000, 25000)

$\tilde{P}$ = (0.095, 0.1, 0.1, 0.105)

Fuzzy optimal order quantity $y_p^*$ **= 1101.97**

TABLE : 1

| S.No | $\lambda$ | $y_p^*$ |
|---|---|---|

| | | |
|---|---|---|
| 1 | 8100 | 1108.23 |
| 2 | 8200 | 1114.55 |
| 3 | 8300 | 1120.98 |
| 4 | 8400 | 1126.57 |
| 5 | 8500 | 1133.13 |

TABLE: 2

| S.No | P | $y_p^*$ |
|---|---|---|
| 1 | 0.11 | 1109.01 |
| 2 | 0.12 | 1116.05 |
| 3 | 0.13 | 1123.16 |
| 4 | 0.14 | 1130.29 |
| 5 | 0.15 | 1137.36 |

TABLE: 3

| S.No | S | $y_p^*$ |
|---|---|---|
| 1 | 25000 | 1103.62 |
| 2 | 26000 | 1105.17 |
| 3 | 27000 | 1106.68 |
| 4 | 28000 | 1108.02 |
| 5 | 29000 | 1109.28 |

## VI. CONCLUSION

This paper proposes fuzzy model of an inventory problem with imperfect items under one-time-only discount. It may be possible and reasonable to discuss the fuzzy inventory models with trapezoidal fuzzy number for crisp order quantity $y_p$, or for fuzzy order quantity $\tilde{y}_p$. The demand, holding cost, ordering cost, purchasing cost, screening rate and defective rate are taken as fuzzy trapezoidal fuzzy number. We find that the optimal fuzzy order quantity $\tilde{y}_p^*$, or the optimal crisp order quantity $y_p^*$ will become

$$\sqrt{\frac{2a\lambda s}{cb\left[s(1-p)^2 + 2\lambda p\right]}}$$

The optimal solution by the proposed method is closer to the crisp solution.

From Table 1 it is observed that the economic order quantity increases when demand increases if all the items are same. From Table 2 it is observed that the economic order quantity increases when the defective percentage increases when all other items are same. From Table 3 it is noticed that the economic order quantity increases when the screening rate increases if all the items are identical .It shows that there is no more increase in the economic order quantity when screening rate increases. The economic order quantity effected due to the increases of demand and defective percentage.

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