Computational Analysis of Multi Server Markovian Queue with Finite Capacity

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***ABSTRACT* - A multi-server Markovian queue with machine repair problem has been considered. If any operating machine fails, it is replaced by a spare machine and the failed machine is admitted to repair. The life time and the repair time of machines are expontially distributed. The balking rule is allowed for failed machines. The values of failure and service rates are changed according to the situation of the system. The management has appointed additional repairman if it is necessary. The computational values for the expressions of the steady state probabilities and expected number of failed machines are obtained. The related curves are exhibited and compared.**

***Keywords* - Multi server queue instantaneous rates, steady state probabilities, expected queue length.**

I. INTRODUCTION

The queueing theory has played and still playing an important role in the theory of probability and related concepts. The applications of queueing theory have been utilized commonly in communication system and industries. Human beings, telephone calls, flow of finished products, failed machines and so on may be considered as queueing units. In modern days, the queueing models have been analysed by assuming the failed machines as the units in the queue for demanding service by means of repair. The failed machines, which may or may not enter the queue depends on the number of already failed machines. The researchers have analysed machine repair problems by using different probability models. Gross and Haris (1985) have described M/M/C/m/m model with spares. Ganesan (1996) has derived the transient state probabilities for general bulk service queue with dependence parameters. Jain (1998) has studied M/M/R machine repair problem with spares and additional repairman. Gupta (1999) has analysed N-policy queueing system with finite source and warm spares. Jain et al (2000) have studied the expected number of failed machines in the system with multiple servers. Shawky (2000) has analysed machine repair problem without additional repairman.

Let λ_n and μ_n be the state dependent arrival and service rates respectively. By providing different functions interms of λ and μ for λ_n and μ_n, many feasible queueing measures have been derived by researchers. These coefficients are first coined by Kawata in 1955. A short list of contributors and their assumptions are summarized below:

- Boes (1969) : $M/M^n/S/N$

$$\lambda_n = \begin{cases} \lambda & , \quad 0 \le n \le s \\ \lambda_r & , \quad s \le n < s+N \\ 0 & , \quad n = s+N \end{cases}$$

$$\mu_n = \begin{cases} n\mu & , 1 \le n \le s-1 \\ s\mu + (n-s)\xi & , \quad s \le n \le s+N \end{cases}$$

(1-r) is the balking probability and ξ is the individual reneging rate.

- Conolly (1975) : $M^n/M/1$

$$\lambda_n = \lambda(n+1)$$
$$\mu_n = n\mu$$

- Natvig (1975) : M/M/1
  a) $\lambda_n = \lambda/(n+1), \mu_n = \mu$
  b) $\lambda_n = \lambda\,(1-\xi n)$

$$\mu_n = \begin{cases} n\nu_n + \Upsilon_n & , \quad 1 \le n \le s \\ s\nu_n + \Upsilon_n & , \quad s < n \le s+N \end{cases}$$

- Ganesan (1996) : $M/M^{a,b}/1$:

$$\lambda_n = \frac{\lambda}{n+1}$$
$$\mu_n = \mu$$

- Shawky (2000) : M/M/C/K/N

$$\lambda_n = \begin{cases} N\lambda & , \ 0 \le n < c \\ N\beta\lambda & , \ c \le n < y \\ (N + y - n)\beta\lambda & , \ y \le n < y + k \\ 0 & , \ n \ge y + k \end{cases}$$

$$\mu_n = \begin{cases} n\mu & , \ 0 \le n < c \\ c\mu + (n - c) \propto & , \ c + 1 \le n \le y + k \end{cases}$$

II. Description of the model

Machine repair problem along with some queueing characters such as balking, spares and additional repairman has been studied. There is a provision that failed machine is replaced immediately if spare machine is available. If all spare machines are used and a machines meets failure, the system becomes short. The lifetime and repair time of the machines are distributed accordingly to exponential distribution. The failed machines are repaired by a group of repairmen. In busy schedule, additional repairman is allowed under the condition that when the number of failed machines in the system is more than 'm' for reducing the backlog of the failed machines and it is removed when the size of the failed machines again reduced to 'm'. After the repair is over, the repaired machines may join along with spares if there is no shortage in the system or may join in the system only when there is some shortage. Here it is assumed that the failure rate of machines and service rate of permanent repairmen depend on the number of failed machines which are waiting for service and are denoted respectively as λ_n and μ_n. The steady-state probabilities for the number of failed machines waiting for service are derived in different situations. The expressions for the expected number of failed machines in the system are derived based on the number of repairmen and number of spare machines. The special cases are studied.

This system is illustrated by the following example: A leading industry has 'N' machines. Out of 'N' machines, d machines kept as spare machines, which are used to replace failed machines. The failed machines have been serviced or repaired by 'c' repairmen. The capacity of the waiting space for the failed machines which are to be repaired is considered as 'k'. Suppose the repair is going on, the number of machines waiting for service is 'k' or more, then the arriving machines may balk. In this busy schedule, even though the spare machines are used instead of failed machines, there is a need to additional server (repairman) for servicing the failed machines, when the waiting space is full or overflow, for avoiding shortages in the production plant. After the repair is over, the machines may either go to the production plant or join with spare machines. It is noted that the concept of reneging is not permitted, since the machines must be serviced for further use. But in the case of human customer or communication system, the reneging policy is possible.

The real life illustration of this concept is the arrival and service of railway engines to the service station. The minor repairs are attended at once but major repairs are done after providing spare machines.

For the purpose of mathematical modeling, the following notations are employed:

k - Capacity of the system

N - Population size of the machines

n - Number of failed machines in the system

d - Number of spare machines

c - Number of permanent repairmen in the system

m - Threshold value of the number of failed machines when additional repairman is on

λ - Failure rate of the machines in the system

β - Balking probability of the machines when all permanent repairmen are busy

μ - Service rate of permanent repairmen

μ_a - Service rate of additional repairman

$P_n(t)$ - Transiant state probability that there are 'n', (n = 0,1,2,….,d+k) failed machines in the system at time 't'

P_n - Steady state probability of n^{th} state

P_0 - Steady state probability of empty state

When all permanent repairmen are busy, the failed machines may balk with probability β. When at least one permanent repairman is free, the balking probability becomes unity.

III. Analysis and Queueing Performance

By the help of λ_n and μ_n, Bharathidass (2007) has discussed a multiple server queue based on machine repair problem and obtained the expressions for steady state probabilities and average queue length in two different cases.

Case (i) $c \le d$

The differential difference equations for the above stated model are framed when the number of repairmen is less than or equal to the number of spare machines and are given below:

$$P_0'(t) = -\lambda_0 P_0(t) + \mu_1 P_1(t) \ ; n = 0 \quad (1)$$

$$P_n'(t) = -(\lambda_n + \mu_n)P_n(t) + \lambda_{n-1}P_{n-1}(t) + \mu_{n+1}P_{n+1}(t); \ 1 \le n < c \quad (2)$$

$$P_c'(t) = -(\lambda_c + \mu_c)P_c(t) + \lambda_{c-1}P_{c-1}(t) + \mu_{c+1}P_{c+1}(t) \ ; n = c \quad (3)$$

$$P_n'(t) = -(\lambda_n + \mu_n)P_n(t) + \lambda_{n-1}P_{n-1}(t) + \mu_{n+1}P_{n+1}(t) \quad ; c+1 \le n \le d$$

………………………….. (4)

$$P_n'(t) = -(\lambda_n + \mu_n)P_n(t) + \lambda_{n-1}P_{n-1}(t) + \mu_{n+1}P_{n+1}(t) \quad ; d+1 \le n < m$$

…………………… (5)

$$P_m'(t) = -(\lambda_m + \mu_m)P_m(t) + \lambda_{m-1}P_{m-1}(t) + \mu_{m+1}P_{m+1}(t) \quad ; n = m \qquad (6)$$

$$P_n'(t) = -(\lambda_n + \mu_n)P_n(t) + \lambda_{n-1}P_{n-1}(t) + \mu_{n+1}P_{n+1}(t) \quad ; m+1 \le n < d+k \qquad (7)$$

$$P_{d+k}'(t) = -\mu_{d+k}P_{d+k}(t) + \lambda_{d+k-1}P_{d+k-1}(t) \quad ; n = d+k \qquad (8)$$

The state dependent failure and service coefficients are defined in different situations.

$$\lambda_n = \begin{cases} N\lambda & , \ 0 \le n < c \\ N\beta\lambda & , \ c \le n < d \\ (N+d-n)\beta\lambda & , d \le n < d+k \\ 0 & , \ n = d+k \end{cases}$$

…………..(9)

and

$$\mu_n = \begin{cases} n\mu & ; \quad 0 < n \le c \\ c\mu & ; \quad c < n \le m \\ c\mu + \mu & ; \quad m < n \le d+k \end{cases} \qquad (10)$$

By applying the equations (9) and (10) in the equations (1) to (8) and adopt some mathematics, the steady state probabilities at different levels are derived as

$$P_n = \begin{cases} \dfrac{\left(N\frac{\lambda}{\mu}\right)^n}{n!} P_0 & ; \quad 0 \le n \le c \\ \dfrac{\left(N\frac{\lambda}{\mu}\right)^n}{c!} \left(\dfrac{\beta}{c}\right)^{n-c} P_0 & ; \quad c+1 \le n \le d \\ \dfrac{N^d}{c!} \left(\dfrac{\lambda}{\mu}\right)^n \left(\dfrac{\beta}{c}\right)^{n-c} N_{(n-d)} P_0 & ; \quad d+1 \le n \le m \\ \dfrac{N^d \beta^{n-c} \lambda^n}{c!\, c^{m-c} \mu^m (c\mu + \mu_a)^{n-m}} N_{(n-d)} P_0 & ; m+1 \le n \le d+k \end{cases}$$

………………………….. (11)

The probability that there is no failed machine in the system (P_0)is estimated by using the normalizing condition.

$$\sum_{n=0}^{c} P_n + \sum_{n=c+1}^{d} P_n + \sum_{n=d+1}^{m} P_n + \sum_{n=m+1}^{d+k} P_n = 1$$

…………… (12)

Substituting the expressions from (11) in equation (12) and get,

$$P_0^{-1} = \sum_{n=0}^{c} \frac{\left[N\left(\frac{\lambda}{\mu}\right)\right]^n}{n!} + \frac{1}{c!}\left(\frac{c}{\beta}\right)^c \sum_{n=c+1}^{d} \left[\frac{N\beta\lambda}{c\mu}\right]^n + \frac{1}{c!}\left(\frac{c}{\beta}\right)^c N^d \sum_{n=d+1}^{m} \left[\frac{\beta\lambda}{c\mu}\right]^n N_{(n-d)}$$

$$+ \frac{1}{c!}\left(\frac{c}{\beta}\right)^c N^d \left(\frac{c\mu + \mu_a}{c\mu}\right)^m \sum_{n=m+1}^{d+k} \left[\frac{\beta\lambda}{c\mu + \mu_a}\right]^n N_{(n-d)} \qquad (13)$$

Based on the equations (11) and (13) the expected number of failed machines in the system is obtained as,

$$L_s = \sum_{n=0}^{d+k} nP_n$$
$$= \left[\sum_{n=1}^{c} \frac{\left[N\left(\frac{\lambda}{\mu}\right)\right]^n}{(n-1)!} + \frac{1}{c!}\left(\frac{c}{\beta}\right)^c \sum_{n=c+1}^{d} n\left[\frac{N\beta\lambda}{c\mu}\right]^n + \frac{1}{c!}\left(\frac{c}{\beta}\right)^c N^d \sum_{n=d+1}^{m} n\left[\frac{\beta\lambda}{c\mu}\right]^n N_{(n-d)} + \frac{1}{c!}\left(\frac{c}{\beta}\right)^c N^d \left(\frac{c\mu+\mu_a}{c\mu}\right)^m \sum_{n=m+1}^{d+k} n\left[\frac{\beta\lambda}{c\mu+\mu_a}\right]^n N_{(n-d)}\right] P_0 \quad (14)$$

Special case

Substituting N = k and $\beta = 1$ in equation (11) they become:

$$P_n = \begin{cases} \frac{\left(\frac{k\lambda}{\mu}\right)^n}{n!} P_0 & ; \quad 0 \le n \le c \\ \frac{\left(\frac{k\lambda}{\mu}\right)^n}{c!} \frac{1}{c^{n-c}} P_0 & ; \quad c+1 \le n \le d \\ \frac{k^d}{c!}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{c^{n-c}} k_{(n-d)} P_0 & ; \ d+1 \le n \le m \\ \frac{k^d \lambda^n}{c!\, c^{m-c}\mu^m (c\mu+\mu_a)^{n-m}} k_{(n-d)} P_0 & ; m+1 \le n \le d+k \end{cases}$$

..........................(15)

where

$$P_0^{-1} = \sum_{n=0}^{c} \frac{\left(\frac{k\lambda}{\mu}\right)^n}{\text{n!}} + \frac{1}{c!} c^c \sum_{n=c+1}^{d} \left(\frac{k\lambda}{c\mu}\right)^n + \frac{1}{c!} c^c k^d \sum_{n=d+1}^{m} \left(\frac{\lambda}{c\mu}\right)^n k_{(n-d)} + \frac{1}{c!} c^c k^d \left(\frac{c\mu+\mu_a}{c\mu}\right)^m \sum_{n=m+1}^{d+k} \left(\frac{\lambda}{c\mu+\mu_a}\right)^n k_{(n-d)}$$

… … … … … … (16)

The expected number of failed machines in the system is

$$L_s = \left[\sum_{n=0}^{c} \frac{\left(\frac{k\lambda}{\mu}\right)^n}{(n-1)!} + \frac{1}{c!} c^c \sum_{n=c+1}^{d} n\left(\frac{k\lambda}{c\mu}\right)^n + \frac{1}{c!} c^c k^d \sum_{n=d+1}^{m} n\left(\frac{\lambda}{c\mu}\right)^n k_{(n-d)} + \frac{1}{c!} c^c k^d \left(\frac{c\mu+\mu_a}{c\mu}\right)^m \sum_{n=m+1}^{d+k} n\left(\frac{\lambda}{c\mu+\mu_a}\right)^n k_{(n-d)}\right] P_0$$

…………………………(17)

Case (ii) c > d

In this case, the number of repairmen is greater than the number of spare machines. Then the differential-difference equations of this model are formed as under:

$$P_0'(t) = -\lambda_0 P_0(t) + \mu_1 P_1(t) \ ; n = 0 \quad (18)$$

$$P_n'(t) = -(\lambda_n + \mu_n) P_n(t) + \lambda_{n-1} P_{n-1}(t) + \mu_{n+1} P_{n+1}(t);\ 1 \le n \le d$$

……………. (19)

$$P_n'(t) = -(\lambda_n + \mu_n) P_n(t) + \lambda_{n-1} P_{n-1}(t) + \mu_{n+1} P_{n+1}(t)\ ; d+1 \le n < c$$

……………… (20)

$$P_c'(t) = -(\lambda_c + \mu_c) P_c(t) + \lambda_{c-1} P_{c-1}(t) + \mu_{c+1} P_{c+1}(t)\ ; n = c$$

……………… (21)

$$P_n'(t) = -(\lambda_n + \mu_n) P_n(t) + \lambda_{n-1} P_{n-1}(t) + \mu_{n+1} P_{n+1}(t) \ \ ; c+1 \le n < m$$

…………………. (22)

$$P_m'(t) = -(\lambda_m + \mu_m) P_m(t) + \lambda_{m-1} P_{m-1}(t) + (\mu_{m+1} + \mu_a) P_{m+1}(t)\ ; n = m$$

……………………. (23)

$$P_n'(t) = -(\lambda_n + \mu_n + \mu_a)P_n(t) + \lambda_{n-1}P_{n-1}(t) + (\mu_{n+1}+\mu_a)P_{n+1}(t); \; m+1 \le n \le d+k \quad \ldots\ldots\ldots (24)$$

Here, the failure and service rates for this model are restated in different situations as follows:

$$\lambda_n = \begin{cases} N\lambda & ; \; 0 \le n < d \\ (N+d-n)\lambda & ; \; d \le n < c \\ (N+d-n)\beta\lambda & ; \; c \le n < d+k \\ 0 & ; \; n = d+k \end{cases} \ldots (25)$$

and

$$\mu_n = \begin{cases} n\mu & ; \; 0 < n \le c \\ c\mu & ; \; c < n \le m \\ c\mu + \mu_a & ; \; m < n \le d+k \end{cases} \ldots (26)$$

By using the equations (25) and (26) in the equations from (18) to (24), the required steady state probabilities at different levels are derived as

$$P_n = \begin{cases} \dfrac{\left[N\left(\frac{\lambda}{\mu}\right)\right]^n}{n!} P_0 & ; \; 0 \le n \le d \\ \dfrac{N^d}{d!}\left(\dfrac{\lambda}{\mu}\right)^n \dfrac{N_{(n-d)}}{c_{(n-d)}} P_0 & ; \; d+1 \le n \le c \\ \dfrac{N^d}{d!}\left(\dfrac{\beta}{c}\right)^{n-c}\left(\dfrac{\lambda}{\mu}\right)^n \dfrac{N_{(n-d)}}{c_{(c-d)}} P_0 & ; \; c+1 \le n \le m \\ \dfrac{N^d}{d!}\dfrac{(\beta\lambda)^n}{(c\mu)^m}\left(\dfrac{c}{\beta}\right)^c \dfrac{1}{(c\mu+\mu_a)^{n-m}}\dfrac{N_{(k)}}{c_{(c-d)}} P_0 & ; \; m+1 \le n \le d+k \end{cases}$$

……………………………. (27)

As in the case of (i), we get,

$$P_0^{-1} = \sum_{n=0}^{d} \frac{N^n}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{N^d}{d!}\sum_{n=d+1}^{c}\left(\frac{\lambda}{\mu}\right)^n \frac{N_{(n-d)}}{c_{(n-d)}} + \frac{N^d}{d!}\left(\frac{c}{\beta}\right)^c \frac{1}{c_{(c-d)}}\sum_{n=c+1}^{m}\left(\frac{\beta\lambda}{c\mu}\right)^n N_{(n-d)} + \frac{N^d}{d!}\left(\frac{c}{\beta}\right)^c \frac{N_{(k)}}{c_{(c-d)}}\left(\frac{c\mu+\mu_a}{c\mu}\right)^m \sum_{n=m+1}^{d+k}\left(\frac{\beta\lambda}{c\mu+\mu_a}\right)^n$$

………………. (28)

The expected number of failed machines in the system is derived by using the expressions (27) and (28).

$$L_s = \sum_{n=0}^{d+k} nP_n = \left[\sum_{n=0}^{d}\frac{N^n}{(n-1)!}\left(\frac{\lambda}{\mu}\right)^n + \frac{N^d}{d!}\sum_{n=d+1}^{c} n\left(\frac{\lambda}{\mu}\right)^n \frac{N_{(n-d)}}{c_{(n-d)}} + \frac{N^d}{d!}\left(\frac{c}{\beta}\right)^c \frac{1}{c_{(c-d)}}\sum_{n=c+1}^{m} n\left(\frac{\beta\lambda}{c\mu}\right)^n + \frac{N^d}{d!}\left(\frac{c}{\beta}\right)^c \frac{N_{(k)}}{c_{(c-d)}}\left(\frac{c\mu+\mu_a}{c\mu}\right)^m \sum_{n=m+1}^{d+k} n\left(\frac{\beta\lambda}{c\mu+\mu_a}\right)^n\right] P_0$$

………………….. (29)

Special case

When $\beta = 1$ and N = k, are applied in the expressions given in (27) which become:

$$P_n = \begin{cases} \dfrac{\left(\frac{k\lambda}{\mu}\right)^n}{n!} P_0 & ; \; 0 \le n \le d \\ \dfrac{k^d}{d!}\left(\dfrac{\lambda}{\mu}\right)^n \dfrac{k_{(n-d)}}{c_{(n-d)}} P_0 & ; \; d+1 \le n \le c \\ \dfrac{k^d}{d!}\dfrac{1}{c^{n-c}}\left(\dfrac{\lambda}{\mu}\right)^n \dfrac{k_{(n-d)}}{c_{(c-d)}} P_0 & ; \; c+1 \le n \le m \\ \dfrac{k^d}{d!}\dfrac{\lambda^n}{(c\mu)^m}\dfrac{c^c}{(c\mu+\mu_a)^{n-m}}\dfrac{k_{(k)}}{c_{(c-d)}} P_0 & ; m+1 \le \mathrm{n} \le \mathrm{d}+\mathrm{k} \end{cases}$$

……………………… (30)

where

$$P_0^{-1} = \sum_{n=0}^{d}\frac{k^n}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{k^d}{d!}\sum_{n=d+1}^{c}\left(\frac{\lambda}{\mu}\right)^n \frac{k_{(n-d)}}{c_{(n-d)}} + \frac{k^d}{d!}\frac{c^c}{c_{(c-d)}}\sum_{n=c+1}^{m}\left(\frac{\lambda}{c\mu}\right)^n k_{(n-d)} + \frac{k^d}{d!}\frac{c^c}{c_{(c-d)}} k_{(k)}\left(\frac{c\mu+\mu_a}{c\mu}\right)^m \sum_{n=m+1}^{d+k}\left(\frac{\lambda}{c\mu+\mu_a}\right)^n \ldots\ldots (31)$$

The expected number of failed machines in the system is

$$L_s = \left[\sum_{n=0}^{d}\frac{k^n}{(n-1)!}\left(\frac{\lambda}{\mu}\right)^n + \frac{k^d}{d!}\sum_{n=d+1}^{c} n\left(\frac{\lambda}{\mu}\right)^n \frac{k_{(n-d)}}{c_{(n-d)}} + \frac{k^d}{d!}\frac{c^c}{c_{(c-d)}}\sum_{n=c+1}^{m} n\left(\frac{\lambda}{c\mu}\right)^n k_{(n-d)} + \frac{k^d}{d!}\frac{c^c}{c_{(c-d)}} k_{(k)}\left(\frac{c\mu+\mu_a}{c\mu}\right)^m \sum_{n=m+1}^{d+k} n\left(\frac{\lambda}{c\mu+\mu_a}\right)^n\right] P_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (32)$$

IV. Computational Analysis and Interpretation

Bharathidass (2007) has formulated a machine repair problem and derived mathematical expressions for the model. He is not attempted to study real life examples. In real life, there are varieties of examples for this model and they are Loco shed in railways for servicing repaired engines, service stations for bus transports, service points for cars, service area for trolleys in industrial campus. For this study, we analysed the activities of repairmen for servicing the trolleys. For this purpose, we select the study area and analyse two service centres of trolleys at Rane Steering Company Private Limited Trichy.

On observing the first centre, there are totally 12 trolleys. There are 3 repairmen and the service centre occupies only 5 trolleys at a time. The management decided to appoint an additional repairman when there are maximum of 8 repaired trolleys are available and kept 5 trolleys under reservation. Based on these information, we collect the number of repaired trolleys at each time, service time for each trolley for regular service and additional service time if additional repairman is appointed. Also collect the number of trolleys balked when all repairmen are busy.

Similarly the second service centre is also observed. They are 6 repairmen for servicing the trolleys and the centre occupies only 8 trolleys at a time. In this centre 2 trolleys only used for spare items and other parameters are treated as in the first centre.

The collected information regarding the service performed for repaired trolleys are used in the expressions for steady state probabilities and expected number failed trolleys for the model designed by Bharathidass (2007) under different conditions.

By assuming different values for λ, the required steady state probabilities and expected number of failed trolleys are presented in the tables from 1 to 5. The corresponding curves are exhibited in the figures from 1 to 5.

The figure 1 to 4 revealed that the steady state probabilities decrease for increasing the arrival rate of failed trolleys. Figure 5 showed that the expected number of failed trolleys increase when the arrival rate of failed trolleys increase. It is also noted that the expected number of failed trolleys decrease when increasing the number of repairmen.

TABLE I
STEADY STATE PROBABILITIES (C$\leq d$)

λ	p_0	p_1	p_2	p_3	p_4
1	2.21E-07	5.30E-06	6.36E-05	0.000509	0.001629
3	5.95E-12	4.28E-10	1.54E-08	3.70E-07	3.55E-06
5	3.89E-14	4.67E-12	2.80E-10	1.12E-08	1.79E-07
7	1.39E-15	2.34E-13	1.96E-11	1.10E-09	2.46E-08
9	1.15E-16	2.48E-14	2.68E-12	1.93E-10	5.55E-09

λ	p_0	p_1	p_2	p_3	p_4
1	8.35E-06	8.35E-05	0.000418	0.001392	0.004641
3	4.23E-10	1.27E-08	1.90E-07	1.90E-06	1.90E-05
5	3.20E-12	1.60E-10	4.00E-09	6.67E-08	1.11E-06
7	1.22E-13	8.53E-12	2.99E-10	6.97E-09	1.63E-07
9	1.04E-14	9.38E-13	4.22E-11	1.27E-09	3.80E-08

TABLE II
STEADY STATE PROBABILITIES (C$\leq d$) WHEN N=K, $\beta = 1$

λ	p_5	p_6	p_7	p_8
1	0.01547	0.051567	0.137512	0.275024
3	0.00019	0.001903	0.015224	0.091344
5	1.85E-05	0.000309	0.004117	0.041167
7	3.79E-06	8.85E-05	0.001653	0.023137
9	1.14E-06	3.42E-05	0.000821	0.014771

λ	p_5	p_6	p_7	p_8
1	0.01547	0.051567	0.137512	0.275024
3	0.00019	0.001903	0.015224	0.091344
5	1.85E-05	0.000309	0.004117	0.041167
7	3.79E-06	8.85E-05	0.001653	0.023137
9	1.14E-06	3.42E-05	0.000821	0.014771

TABLE III
STEADY STATE PROBABILITIES (C$> d$)

λ	p_0	p_1	p_2	p_3	p_4
1	8.81E-07	2.11E-05	0.000254	0.001014	0.004057
3	5.94E-11	4.28E-09	1.54E-07	1.85E-06	2.22E-05
5	5.33E-13	6.40E-11	3.84E-09	7.68E-08	1.54E-06
7	2.27E-14	3.82E-12	3.21E-10	8.99E-09	2.52E-07
9	2.11E-15	4.55E-13	4.91E-11	1.77E-09	6.37E-08

λ	p_5	p_6	p_7	p_8
1	0.02231	0.1339	0.1428	0.1333
3	0.000366	0.006587	0.02108	0.05902
5	4.22E-05	0.001266	0.006755	0.03152
7	9.69E-06	0.000407	0.003037	0.01984
9	3.15E-06	0.00017	0.001634	0.01373

TABLE IV
STEADY STATE PROBABILITIES (C> d) WHEN N=K,$\beta = 1$

λ	p_0	p_1	p_2	p_3
1	2.12E-05	0.00034	0.002718	0.005436
3	2.37E-09	1.14E-07	2.73E-06	1.64E-05
5	2.18E-11	1.74E-09	6.96E-08	6.96E-07
7	9.01E-13	1.01E-10	5.65E-09	7.92E-08
9	8.08E-14	1.16E-11	8.38E-10	1.51E-08

p_4	p_5	p_6	p_7	p_8
0.01087	0.04349	0.174	0.2609	0.2706
9.83E-05	0.00118	0.01416	0.06372	0.1983
6.96E-06	0.000139	0.002784	0.02088	0.1083
1.11E-06	3.10E-05	0.000869	0.009122	0.06622
2.71E-07	9.77E-06	0.000352	0.004748	0.04432

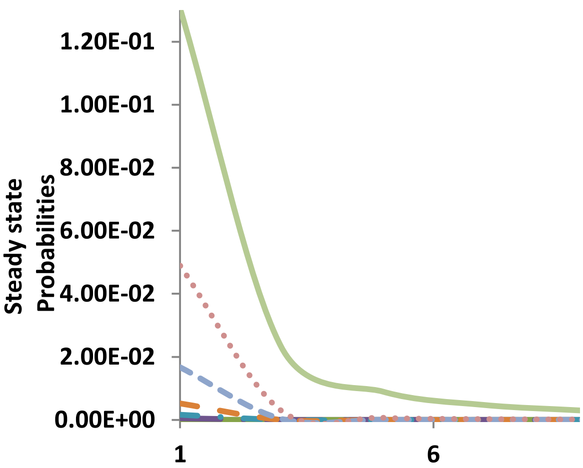

Fig. 1

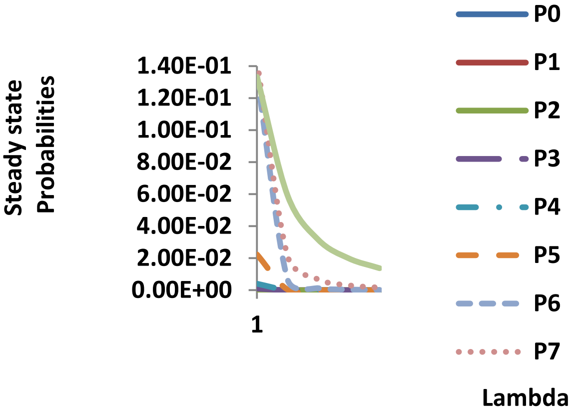

Fig. 2

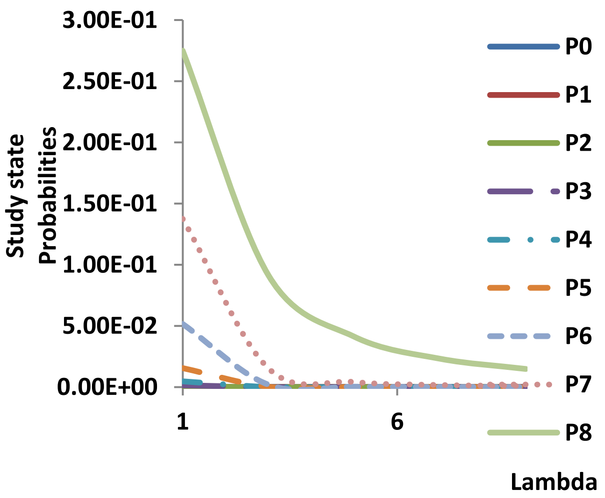

Fig. 3

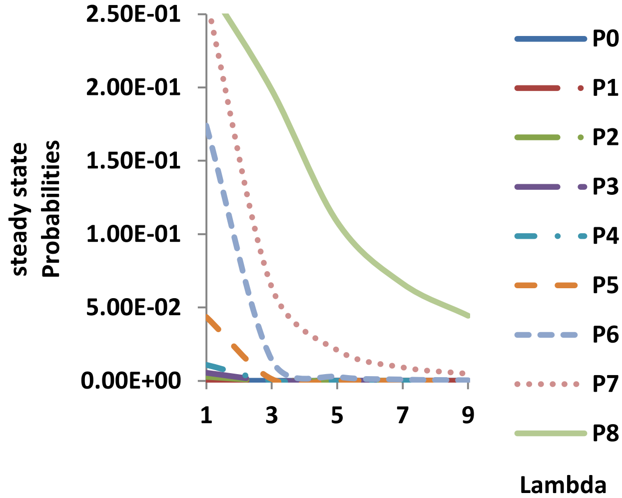

Fig. 4

TABLE V
EXPECTED NUMBER OF FAILED TROLLEYS(L_s)

λ	Ls1 (c≤ d)	Ls2 ($c \leq d$) (N=k,β = 1)	Ls3 (c> d)	Ls4 (c> d) (N=k,β = 1)
1	9.20921	8.38847	8.124	7.436
3	9.79725	9.44056	9.126	8.970
5	9.88470	9.66182	9.320	9.168
7	9.91953	9.75785	9.429	9.246
9	9.93822	9.81146	9.504	9.346

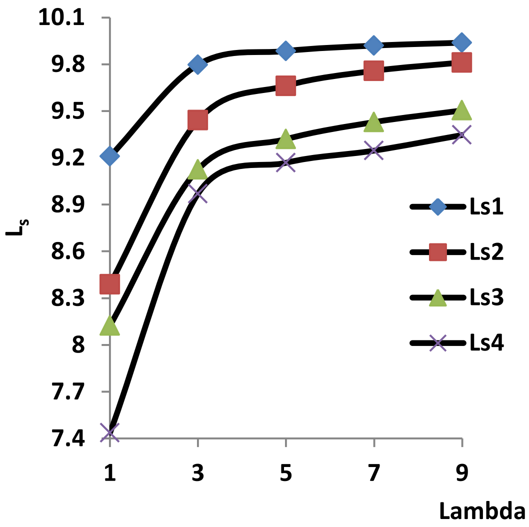

Fig. 5

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