Numerical Studies on Fuzzy Retrial Queues

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***ABSTRACT* – This paper aims to construct the membership function for fuzzy retrial queue system using non-linear programming approach with three fuzzy variables, fuzzified exponential arrival, retrial and service rate which are represented as fuzzy numbers. Using α-cut approach, fuzzy retrial queues can be reduced to a family of crisp retrial queues with different α-cuts. Triangular fuzzy numbers are used to demonstrate the validity of the proposal. The discussion of this paper is confined to systems with one two fuzzy variables: nevertheless, the procedure can be extended to systems with more than two fuzzy variables. Numerical illustration has been carried out successfully.**

***Keywords* - Orbit, Triangular Fuzzy umber, Membership functions, Retrial queues and Parametric Programming**

I. INTRODUCTION

In retrial queue system, an arriving customer who finds the server busy is obliged to leave the service area and join a pool of unsatisfied customers called the **"Orbit".** From the orbit, each customer further applies for the service after an uncertain amount of time called **Retrial time.**

There are so many applications found for retrial queues in science and engineering streams. Recently Diamond and Alfa [3] constructed a method for approximating the stationary distributions and waiting time moments of M/PH/1 retrial queue with phase inter-retrial times. The BMAP/G/1 retrial system with search for customers immediately on termination of service was studied by Dudin et al [1], in which inter-retrial is followed by exponential distribution and duration of search is characterized by a generally distributed variable. Lopez-Herrero [5] presented the explicit formulae for the probabilities of the number of customers being served in a busy period and an explicit expression for the second moment for M/G/1 retrial queueing system has also been given. Chuen-Horng Lin, Jau-Chuan, Hsin Huang [2] constructed the membership function for fuzzy retrial queueing system using Trapezoidal fuzzy number with the concept of non-linear programming approach with three fuzzy variables, fuzzified exponential arrival, retrial and service rate. The model in notation form can be denoted by FM/FM/1/1-FR where F denotes the fuzzified exponential rate with single server and single system capacity. The fuzzy expected waiting time in the system for retrial system as well as expected number of customers in the system has been presented through α-cut and graphs. α-cut approach is used to construct system characteristic membership function.

II. FUZZY SET THEORY [4]

Definition.2.1. In the universe of discourse X, a fuzzy subset Ã on X is defined by the membership function $\mu_{\tilde{A}}(X)$ which maps each element x in X to a real number in the interval [0, 1]. $\mu_{\tilde{A}}(X)$ denotes the grade or degree of membership and it is usually denoted as

$\mu_{\tilde{A}}(X)$:X→ [0, 1].

Definition.2.2. The support of a fuzzy set Ã is the crisp set such that it is represented as

Supp Ã(X) = $\{x \in X / \mu_{\tilde{A}}(x) \succ 0\}$.

Definition.2.3. The height of fuzzy set Ã is h(Ã) = $\sup_{x\in X} \mu_{\tilde{A}}(x)$. ie, the least upper bound of $\mu_{\tilde{A}}(X)$.

Definition.2.4. A fuzzy set Ã is said to be normalized iff there exist $x \in X$, such that $\mu_{\tilde{A}}(X)$=1.

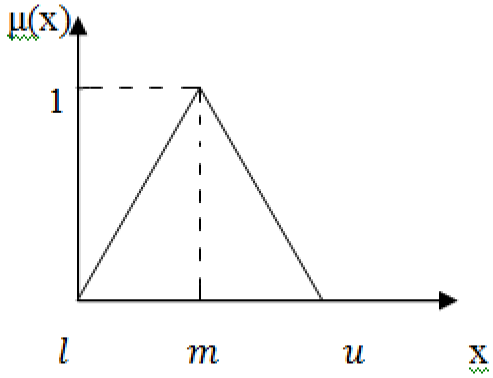
i.e, h(Ã) = 1. If Ã is not normal, then it is called subnormal.

III. TRIANGULAR FUZZY NUMBER

We define a fuzzy number M on R to be a triangular fuzzy number if its membership function $\mu_M(x): R \rightarrow [0,1]$ is defined by

$$\mu_M(x) = \begin{cases} \dfrac{x-l}{m-l}, & \text{for} \quad l \le x \le m \\ \dfrac{x-u}{m-u}, & \text{for} \quad m \le x \le u \\ 0, & \text{otherwise} \end{cases}$$

where $l \le m \le u$, l and u stand for the lower and upper value of the support of M respectively and m for the modal value. The triangular fuzzy number can be denoted by (l, m, u). The support of M is the set of elements $\{x \in R / l \prec x \prec u\}$, when $l = m = u$, it is a non-fuzzy number by convention. Here the vertical line shows the membership function.



IV. FUZZY RETRIAL QUEUES

In this model, the fuzzy arrival rate $\tilde{\lambda}$, fuzzy retrial rate $\tilde{\gamma}$ and fuzzy service rate $\tilde{\mu}$ can be represented as convex fuzzy sets. Let $\mu_{\tilde{\lambda}}(\tilde{\lambda})$, $\mu_{\tilde{\gamma}}(\tilde{\gamma})$, $\mu_{\tilde{\theta}}(\tilde{\theta})$ be the membership functions of $\tilde{\lambda}, \tilde{\gamma}$ and $\tilde{\theta}$ respectively. The fuzzy sets for the above rates are defined by

$$\tilde{\lambda} = \{(x, \mu_{\tilde{\lambda}}(x)) / x \in S(\tilde{\lambda})\}$$

$$\tilde{\gamma} = \{(y, \mu_{\tilde{\gamma}}(y)) / y \in S(\tilde{\gamma})\}$$

$$\tilde{\theta} = \{(z, \mu_{\tilde{\theta}}(z)) / z \in S(\tilde{\theta})\}$$

where X = $S(\tilde{\lambda})$, Y = $S(\tilde{\gamma})$ and Z = $S(\tilde{\theta})$ are the supports of $\tilde{\lambda}$, $\tilde{\gamma}$ and $\tilde{\theta}$ which are the crisp universal sets of arrival rate, retrial rate and service rate respectively. Clearly when $\tilde{\lambda}, \tilde{\gamma}$ and $\tilde{\theta}$ are fuzzy numbers then the performance measure $\tilde{p}(\tilde{\lambda}, \tilde{\gamma}, \tilde{\theta})$ are also fuzzy as well. On the basis of Zadeh's extension principle (6,7) the membership function of the performance measure is defined as

$$\mu_{\tilde{p}(\tilde{\lambda},\tilde{\gamma},\tilde{\theta})}(z) = \sup_{x \in X, y \in Y, z \in Z} \min \{(\mu_{\tilde{\lambda}}(x), \mu_{\tilde{\gamma}}(y), \mu_{\tilde{\theta}}(z)) / z = p(x, y, z)\}$$

Assume that the system characteristic of interest is the expected waiting time $\tilde{W}$ =E(W) and expected number of customers $\tilde{L}$ = E(N) in the system. From the classical queueing theory, if $\dfrac{\lambda}{\theta} \prec 1$, then it becomes steady-state condition under which, the expected number of customers in the orbit is defined to be

$$N_o = \frac{\rho^2}{1-\rho} * \frac{\vartheta + \gamma}{\gamma} \qquad \text{………(4.1)}$$

and the expected waiting time in the orbit is defined by

$$W_o = \frac{\rho}{\theta - \lambda} * \frac{\theta + \gamma}{\gamma} \quad \text{……….(4.2)}$$

The expected waiting time in the system is defined to be

W =E(W) = $W_o + \dfrac{1}{\theta}$

= $\dfrac{\rho}{\theta - \lambda} * \dfrac{\theta + \gamma}{\gamma} + \dfrac{1}{\theta}$,

where $\rho = \dfrac{\lambda}{\theta}$

Therefore W =E(W) = $\left(\dfrac{1}{\theta - \lambda}\right)\left(\dfrac{\lambda + \gamma}{\gamma}\right)$

Using Little's formula,

E(N) = $L = \lambda W = \left(\dfrac{\lambda}{\theta - \lambda}\right)\left(\dfrac{\lambda + \gamma}{\gamma}\right)$

In FM/FM/1/1-R (Fuzzy retrial queue), the expected waiting time and the expected number of customers in the system respectively given by

$$\tilde{W} = \left(\frac{1}{\tilde{\theta} - \tilde{\lambda}}\right)\left(\frac{\tilde{\lambda} + \tilde{\gamma}}{\tilde{\gamma}}\right) \quad \text{………(4.3)}$$

and

$$\tilde{L} = \left(\frac{\tilde{\lambda}}{\tilde{\theta} - \tilde{\lambda}}\right)\left(\frac{\tilde{\lambda} + \tilde{\gamma}}{\tilde{\gamma}}\right) \quad \text{………..(4.4)}$$

V. The Solution Procedure

One approach to construct the membership function of $\mu_{\tilde{p}(\tilde{\lambda},\tilde{\gamma},\tilde{\theta})}(z)$ is on the basis of deriving α-cuts of $\tilde{\lambda}, \tilde{\gamma}$ and $\tilde{\theta}$. Denote α-cuts of $\tilde{\lambda}, \tilde{\gamma}$ and $\tilde{\theta}$ as

$$\tilde{\lambda}_\alpha = \left[x_\alpha{}^L, x_\alpha{}^U\right] = \left\{\min_{x\in X}\left[x / \mu_{\tilde{\lambda}}(x) \geq \alpha\right], \max_{x\in X}\left[x / \mu_{\tilde{\lambda}}(x) \geq \alpha\right]\right\} \quad \text{……………………(5.1)}$$

$$\tilde{\gamma}_\alpha = \left[y_\alpha{}^L, y_\alpha{}^U\right] = \left\{\min_{y\in Y}\left[y / \mu_{\tilde{\gamma}}(y) \geq \alpha\right], \max_{y\in Y}\left[y / \mu_{\tilde{\gamma}}(y) \geq \alpha\right]\right\} \quad \text{…………………...(5.2)}$$

$$\tilde{\theta}_\alpha = \left[z_\alpha{}^L, z_\alpha{}^U\right] = \left\{\min_{z\in Z}\left[z / \mu_{\tilde{\theta}}(z) \geq \alpha\right], \max_{z\in Z}\left[z / \mu_{\tilde{\theta}}(z) \geq \alpha\right]\right\} \quad \text{…………………...(5.3)}$$

These intervals indicate where the group arrival rate, retrial rate and service rate lie at possibility level α. Consequently, the fuzzy retrial queues can be reduced to a family of crisp retrial queues with different α-level sets $\tilde{\lambda}_\alpha, \tilde{\gamma}_\alpha$ and $\tilde{\theta}_\alpha$, where 0 < α ≤ 1.

By the convexity of a fuzzy number [7], the bands of these intervals are functions of α and can be obtained as

$x_\alpha{}^L$ = min $\mu_{\tilde{\lambda}}{}^{-1}(\alpha)$ and $x_\alpha{}^U$ = max $\mu_{\tilde{\lambda}}{}^{-1}(\alpha)$

$y_\alpha{}^L$ = min $\mu_{\tilde{\gamma}}{}^{-1}(\alpha)$ and $y_\alpha{}^U$ = max $\mu_{\tilde{\gamma}}{}^{-1}(\alpha)$

$z_\alpha{}^L$ = min $\mu_{\tilde{\theta}}{}^{-1}(\alpha)$ and $z_\alpha{}^U$ = max $\mu_{\tilde{\theta}}{}^{-1}(\alpha)$

To find the membership function $\mu_{\tilde{l}}(z)$, we have to find the lower bound $l_{L(\alpha)}$ and the upper bound $u_{L(\alpha)}$ of α-cuts of $\mu_{\tilde{l}}(z)$. Since the requirement of $\mu_{\tilde{\lambda}}(x) = \alpha$ can be represented by $x = x_\alpha{}^L$ or $x = x_\alpha{}^U$ this can be formulated as the constraint of $x = \beta_1 x_\alpha{}^L + (1-\beta_1)x_\alpha{}^U$,where $\beta_1 = 0$ (or) 1.Similarly $\mu_{\tilde{\gamma}}(y) = \alpha$ can be formulated as the constraint $y = \beta_2 y_\alpha{}^L + (1-\beta_2)y_\alpha{}^U$,
where $\beta_2 = 0$ (or) 1 and $\mu_{\tilde{\theta}}(z) = \alpha$ can be formulated as the constraint $z = \beta_3 z_\alpha{}^L + (1-\beta_3)z_\alpha{}^U$,
where $\beta_3 = 0$ (or) 1 .The parametric programming problem has the following form

$$\left.\begin{aligned} & l_{p(\alpha)} = \min p(x,y,z) \\ \text{such that} \quad & l_{\lambda_\alpha} \leq x \leq u_{\lambda_\alpha} \\ & l_{\gamma_\alpha} \leq y \leq u_{\gamma_\alpha} \\ & l_{\theta_\alpha} \leq z \leq u_{\theta_\alpha} \end{aligned}\right\} \quad \text{…………………………….(5.4)}$$

and

$$\left.\begin{array}{ll} & u_{p(\alpha)} = \max\ p(x, y, z) \\ \text{such that} & l_{\lambda_\alpha} \le x \le u_{\lambda_\alpha} \\ & l_{\gamma_\alpha} \le y \le u_{\gamma_\alpha} \\ & l_{\theta_\alpha} \le z \le u_{\theta_\alpha} \end{array}\right\} \qquad \text{.................................(5.5)}$$

More over from the definition of (5.1),(5.2) and (5.3), $x \in \lambda_\alpha$, $y \in \gamma_\alpha$ and $z \in \theta_\alpha$ respectively replaced by $x \in (x_\alpha{}^L, x_\alpha{}^U)$, $y \in (y_\alpha{}^L, y_\alpha{}^U)$ and $z \in (z_\alpha{}^L, z_\alpha{}^U)$ .Consequently, considering all these three cases, the membership function $\mu_{\tilde{l}}(z)$ can be constructed via finding the lower bound $l_{L(\alpha)}$ and the upper bound $u_{L(\alpha)}$.

If both $l_{L(\alpha)}$ and $u_{L(\alpha)}$ are invertible with respect to α, then a left shape function $L_s(z) = \left(l_{L(\alpha)}\right)^{-1}$ and right shape function $R_s(z) = \left(u_{L(\alpha)}\right)^{-1}$ can be obtained. From $L_s(z)$ and $R_s(z)$ the membership function $\mu_{\tilde{l}}(z)$ can be constructed as follows

$$\mu_{p(\tilde{\lambda},\tilde{\gamma},\tilde{\theta})}(z) = \begin{cases} L_s(z), & z_1 \le z \le z_2 \\ R_s(z), & z_2 \le z \le z_3 \\ 0, & \text{otherwise} \end{cases} \qquad \text{.........................(5.6)}$$

where $z_1 \le z_2 \le z_3$, $L(z_1) = R(z_3) = 0$. Since the above performance measures are described by membership functions, they conserve completely all fuzziness of arrival rate, retrial rate and service rate.

VI. Illustration

Consider a centralized parallel processing in which the arrival rate, retrial rate and service rate are triangular fuzzy numbers represented by $\tilde{\lambda}$ = (7, 8, 9), $\tilde{\gamma}$ = (2, 3, 4) and $\tilde{\theta}$ = (14, 15, 16) per minute respectively. The system manager wants to evaluate the performance measures of the system such as expected number of customers and waiting time of the customers in the system.

The α-cuts of arrival rate $\tilde{\lambda}$, retrial rate $\tilde{\gamma}$ and service rate $\tilde{\theta}$ are respectively given by

$$\tilde{\lambda}_\alpha = [7+\alpha\ ,\ 9-\alpha\ ],\ \tilde{\gamma}_\alpha = [2+\alpha\ ,\ 4-\alpha\ ] \text{ and } \tilde{\theta}_\alpha = [14+\alpha\ ,\ 16-\alpha\ ].$$

From equations (5.4) and (5.5), the parametric programming problem are formulated to derive the membership functions of $\tilde{L}$, the number of customers and $\tilde{W}$, the waiting time of the customers in the system. They are calculated as follows using (5.4) and (5.5), but they differ only in their objective functions.

$$\left.\begin{array}{ll} \text{(i).} \quad l_{L(\alpha)} = & \min\left\{\left(\dfrac{\tilde{\lambda}}{\tilde{\theta}-\tilde{\lambda}}\right)\left(\dfrac{\tilde{\lambda}+\tilde{\gamma}}{\tilde{\gamma}}\right)\right\} \\ & 7+\alpha \le \tilde{\lambda} \le 9-\alpha \\ \text{such that} & 2+\alpha \le \tilde{\gamma} \le 4-\alpha \\ & 14+\alpha \le \tilde{\theta} \le 16-\alpha \end{array}\right\} \qquad \text{................................(6.1)}$$

$$\left.\begin{array}{ll} u_{L(\alpha)} = & \max\left\{\left(\dfrac{\tilde{\lambda}}{\tilde{\theta}-\tilde{\lambda}}\right)\left(\dfrac{\tilde{\lambda}+\tilde{\gamma}}{\tilde{\gamma}}\right)\right\} \\ & 7+\alpha \le \tilde{\lambda} \le 9-\alpha \\ \text{such that} & 2+\alpha \le \tilde{\gamma} \le 4-\alpha \\ & 14+\alpha \le \tilde{\theta} \le 16-\alpha \end{array}\right\} \qquad \text{................................(6.2)}$$

where $0 < \alpha \leq 1$.

$l_{L(\alpha)}$ is found when $\tilde{\lambda}$ and $\tilde{\theta}$ in the first bracket and $\tilde{\lambda}$ and $\tilde{\gamma}$ in the second bracket of the objective function approach their lower bounds and upper bounds of the α-cuts respectively. Consequently the optimal solution for (6.1) is

$$l_{L(\alpha)} = \frac{91-\alpha-2\alpha^2}{28-7\alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6.3)$$

Also $u_{L(\alpha)}$ is found when $\tilde{\lambda}$ and $\tilde{\theta}$ in the first bracket and $\tilde{\lambda}$ and $\tilde{\gamma}$ in the second bracket of the objective function approach their upper bounds and lower bounds of the α-cuts respectively. Then the optimal solution for (6.2) is

$$u_{L(\alpha)} = \frac{81+9\alpha-2\alpha^2}{14+7\alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6.4)$$

The membership function $\mu_{\tilde{l}}(z)$ is obtained as

$$\mu_{\tilde{l}}(z) = \begin{cases} L(z), & \lfloor l_{L(\alpha)} \rfloor_{\alpha=0} \leq z \leq \lfloor l_{L(\alpha)} \rfloor_{\alpha=1} \\ R(z), & \lfloor u_{L(\alpha)} \rfloor_{\alpha=1} \leq z \leq \lfloor u_{L(\alpha)} \rfloor_{\alpha=0} \\ 0 & \text{otherwise} \end{cases}$$

which is estimated as

$$\mu_{\tilde{L}}(z) = \begin{cases} \dfrac{(7z-1)-(49z^2-238z+729)^{\frac{1}{2}}}{4}, & 3.25 \leq z \leq 4.1905 \\ \dfrac{-(7z-9)+(49z^2-238z+729)^{\frac{1}{2}}}{4}, & 4.1905 \leq z \leq 5.7857 \\ 0 & \text{otherwise} \end{cases} \quad \ldots\ldots\ldots\ldots..(6.5)$$

Similarly, the performance functions of $\tilde{W}$, the waiting time of the customers in the system can be derived from the respective parametric programs which differ only in their objective function.

$$\text{(ii).} \quad l_{W(\alpha)} = \min\left\{\left(\frac{1}{\tilde{\theta}-\tilde{\lambda}}\right)\left(\frac{\tilde{\lambda}+\tilde{\gamma}}{\tilde{\gamma}}\right)\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(6.6)$$

and

$$u_{W(\alpha)} = \max\left\{\left(\frac{1}{\tilde{\theta}-\tilde{\lambda}}\right)\left(\frac{\tilde{\lambda}+\tilde{\gamma}}{\tilde{\gamma}}\right)\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6.7)$$

yield the following results

$$l_{W(\alpha)} = \frac{13-2\alpha}{28-7\alpha} \quad ; \quad u_{W(\alpha)} = \frac{9+2\alpha}{14+7\alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6.8)$$

with their membership function as

$$\mu_{\tilde{W}}(z) = \begin{cases} \dfrac{(28z-13)}{(7z-2)}, & 0.4643 \leq z \leq 0.5238 \\ \dfrac{-(14z-9)}{(7z-2)} & 0.5238 \leq z \leq 0.6429 \\ 0 & \text{otherwise} \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots...(6.9)$$

Table I. refers to α-cuts of arrival rate, retrial rate and service rate, lower and upper bounds of them. Fig: (1) and Fig: (2) represent the graphs of the membership functions of the expected number of customers and waiting time of the customers in the system.

TABLE I
A-CUTS OF ARRIVAL RATE, RETRIAL RATE, SERVICE RATE FOR FUZZY RETRIAL QUEUE (15- A-CUTS)

α	$l_{\tilde{\lambda}(\alpha)}$	$u_{\tilde{\lambda}(\alpha)}$	$l_{\tilde{\gamma}(\alpha)}$	$u_{\tilde{\gamma}(\alpha)}$	$l_{\tilde{\mu}(\alpha)}$	$u_{\tilde{\mu}(\alpha)}$	$l_{E(\tilde{W})}$	$u_{E(\tilde{W})}$	$l_{E(\tilde{N})}$	$u_{E(\tilde{N})}$
0	7.0	9	2.0	4.0	14.0	16.0	0.3125	0.7347	2.1875	6.6122
0.05	7.05	8.95	2.05	3.95	14.05	15.95	0.3181	0.7147	2.2427	6.3965
0.1	7.1	8.9	2.1	3.9	14.1	15.9	0.3239	0.6956	2.2994	6.1910
0.2	7.2	8.8	2.2	3.8	14.2	15.8	0.3358	0.6600	2.4176	5.8076
0.25	7.25	8.75	2.25	3.75	14.25	15.75	0.3420	0.6433	2.4791	5.6287
0.3	7.3	8.7	2.3	3.7	14.3	15.7	0.3483	0.6273	2.5424	5.4574
0.4	7.4	8.6	2.4	3.6	14.4	15.6	0.3614	0.5972	2.6745	5.1361
0.5	7.5	8.5	2.5	3.5	14.5	15.5	0.3753	0.5695	2.8144	4.8404
0.55	7.55	8.45	2.55	3.45	14.55	15.45	0.3824	0.5564	2.8874	4.7012
0.6	7.6	8.4	2.6	3.4	14.6	15.4	0.3898	0.5437	2.9627	4.5673
0.7	7.7	8.3	2.7	3.3	14.7	15.3	0.4052	0.5198	3.1203	4.3145
0.75	7.75	8.25	2.75	3.25	14.75	15.25	0.4133	0.5085	3.2028	4.1949
0.8	7.8	8.2	2.8	3.2	14.8	15.2	0.4215	0.4975	3.2879	4.0796
0.9	7.9	8.1	2.9	3.1	14.9	15.1	0.4388	0.4767	3.4665	3.8611
1.0	8.0	8.0	3.0	3.0	15.0	15.0	0.4571	0.4571	3.6571	3.6571

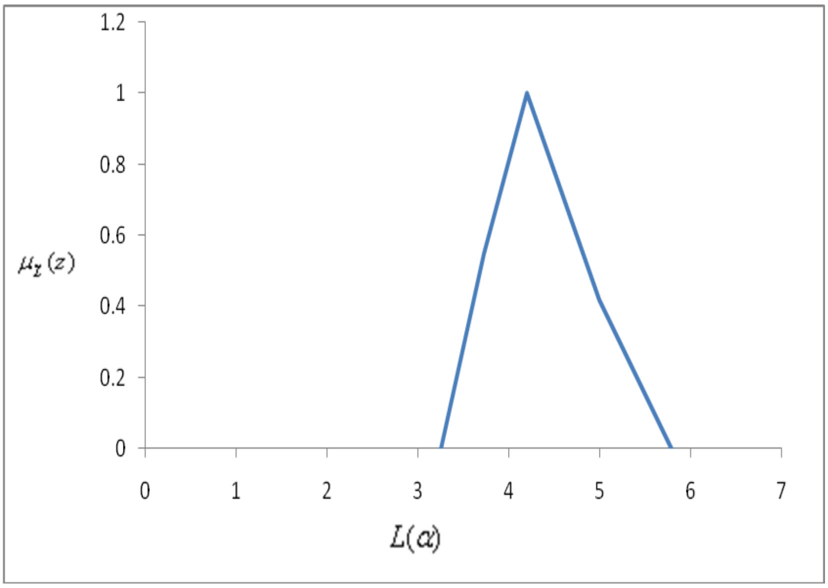


Fig. 1 Expected Number of Customers in the System

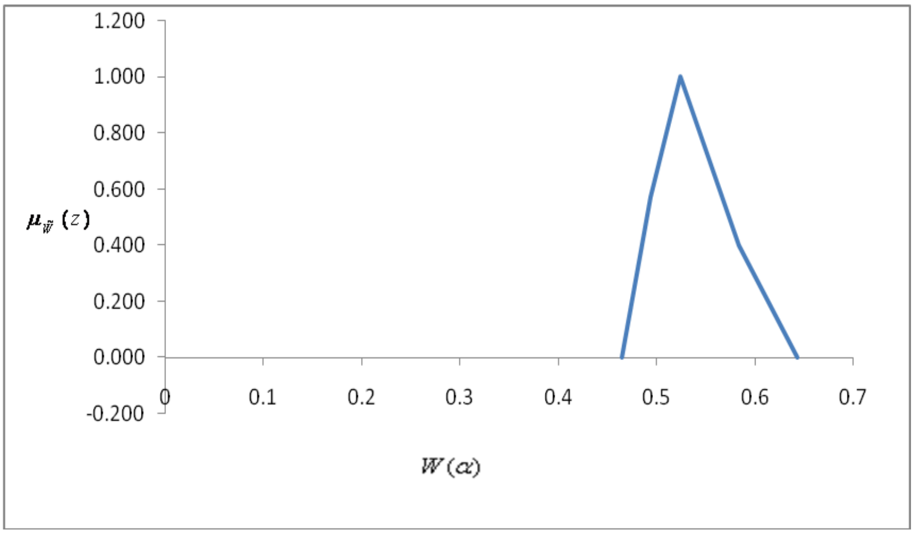


Fig. 2 Average Waiting Time of The Customers in the Orbit

VII. CONCLUSION

In this paper, we have studied system characteristic of retrial queueing system under fuzzy environment. The fuzzy triangular number is being used to derive the system characteristic of fuzzy retrial queue model. The waiting time as well as expected number of customers in the system has been computed by using non-linear parametric programming approach. The proposed model is more realistic and more suitable for designer and practitioners. α-cut approach and parametric programming are used to construct system characteristic membership function for preserving the fuzziness.

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